DETERMINATION OF ULTIMATE SWEEP EFFICIENCY IN A LINEAR WATER FLOOD

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ABSTRACT

Primary recovery of oil is that mode of production during which the reservoir pressure is sufficient to overcome the hydraulic pressure head in the wellbore and, thus, allow the fluids to reach the surface. Often, more than 80% of the original-oil-in-place remains in the ground following this phase of recovery. Fluid, injected in wells adjacent to producers, displaces or "sweeps" the oil.[1]

Secondary recovery of oil consists of either the displacement by gas or water. The natural dip angle of the producing reservoir has a major effect on the sweep efficiency.[1]

This paper presents two new techniques developed to determine the ultimate sweep efficiency in a water flood. The first technique is analytic while the second is a numerical approximation. A reservoir model is also scaled and used to vindicate the predictive equations.

Introduction

The developments in this paper to determine the ultimate sweep efficiency varies from previous attempts in that they both require relative permeability data at residual oil saturations $(\frac{k_w}{k})$, instead of complex and often unreliable laboratory experiments over the whole range of oil saturations during a water flood operation.[2]

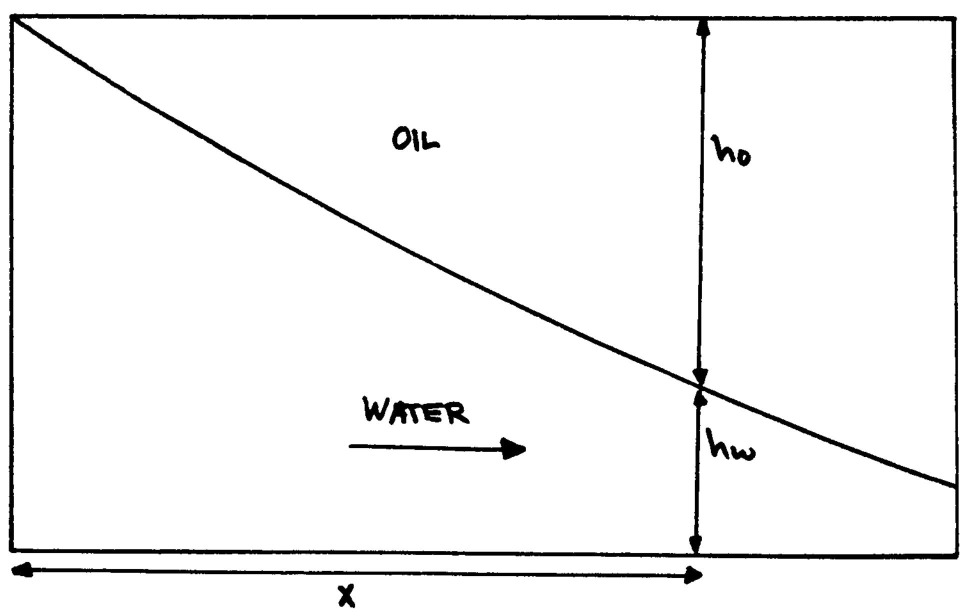
Knowledge of the final sweep efficiency and profile help the engineer to decide on optimum flow rate and also the most suitable level in the payzone to produce from to prevent excessive water production in the latter phases of the waterflood operation.

Theory

Analytic Technique

Buckley-Leveritt[3] converts the two dimensional problem in a linear reservoir to one dimension by taking an average saturation over the thickness of the reservoir. In this paper's technique Hubberts potential[2] and Darcy's Law[4] are used to allow a two dimensional system to be described. Buckely-Leveritt's model allows a description in relation to time, but this method is essentially static. (See Figure 1)

Figure 1



Darcy's Law: $$q_w = - \frac{kh_w}{\mu} \rho_w \frac{d\Phi}{dl} \qquad (1)$$

Hubberts Potential: $$\Phi = \frac{P}{\rho_w} + gz \qquad (2)$$

Take a horizontal reservoir: $gz = 0$

$$P = \rho_w h_w g + \rho_o h_o g + \rho_w heg \qquad (3)$$

he = driving pressure (Height of water equivalent)

$$h_o = h - h_w \qquad (4)$$

Replace h_o in (3) by (4).

$$P = \rho_w h_w g + \rho_o hg - \rho_o h_w g + \rho_w heg \qquad (5)$$

Substitute (5) into (2).

$$\Phi = \frac{g}{\rho_w} [h_w (\rho_w - \rho_o) + \rho_o h + \rho_w he]$$

$$\Phi = gh_w \frac{(\rho_w - \rho_o)}{\rho_w} + \frac{\rho_o}{\rho_w} hg + heg$$

$$\frac{d\Phi}{dx} = \frac{dh_w}{dx} g \frac{(\rho_w - \rho_o)}{\rho_w} + g \frac{dhe}{dx} \qquad (6)$$

Substitute 6 into 1.

$$q = - \frac{kh_w g (\rho_w - \rho_o)}{\mu} \frac{dh_w}{dx} - \frac{kh_w \rho_w g}{\mu} \frac{dhe}{dx} \qquad (7)$$

Now h_w varies along the length of the reservoir, q remains constant, therefore the velocity of the fluid also changes, therefore dhe/dx increases as h_w reduces. Assume the rate of change of $\frac{dh_w}{dx}$ along the reservoir is negligible, therefore $\frac{dhe}{dx}$ must increase by h/h_w. Substitute h/h_w into equation 7.

$$q = - \frac{kh_w \; g \; \rho_w - \rho_o}{\mu} \frac{dh_w}{dx} - \frac{k\rho_w gh}{\mu} \frac{dhe}{dx} \tag{8}$$

$$C_1 = \frac{kg \; (\rho_w - \rho_o)}{\mu}$$

$$C_2 = \frac{khg\rho_w}{\mu} \frac{dhe}{dx} \qquad \text{het} \quad H = \frac{dhe}{dx}$$

$$q = - C_1 h_w \frac{dh_w}{dx} - C_2$$

$$C_1 h_w \frac{dh_w}{dx} = - C_2 - q$$

Separate variables, integrate and solve for h_w.

$$\int_h^{h_w} h_w dh_w = \left(- \frac{C_2}{C_1} - \frac{q}{C_1}\right) \int_0^x dx$$

$$\frac{h_w^2}{2} - \frac{h^2}{2} - \left(- \frac{C_2}{C_1} - \frac{q}{C_1}\right) x$$

$$h_w = 2 \left(- \frac{C_2}{C_1} - \frac{q}{C_1}\right) x + h^2$$

$$h_w = 2 \left(\frac{q\mu}{kg \; (\rho_o - \rho_w)} + \frac{h\rho_w H}{(\rho_o - \rho_w)}\right) x + h^2 \tag{9}$$

Dimensional Analysis

$$\frac{C_2}{C_1} x = \frac{LMLL^3}{L^3M} = L^2$$

$$\frac{qx}{C_1} - \frac{L^2M\ T^2L^3L}{TLTL^2LM} = L^2$$

Note: The units of q are $\frac{L^2}{T}$ as we are using h_w as equivalent to area in Darcy's Formula (Equation 1).

Numerical Technique

This method assumes similar circumstances as for the analytic technique.

i In the latter flood stages only one phase is considered to flow (water).

ii Horizontal reservoir.

iii Homogeneous reservoir.

iv Immiscible fluid.

v Negligible capillary forces (See scaling section).

v Gravity effects are the main influence on fluid flow.

vi Constant flow rate.

v Unit width.

Assume the reservoir is producing only water and the profile is identical to Figure 1 due to gravitational forces causing the less dense oil to ride above the water.

Now separate the crosssection in vertical sections of length Δx and height h_w.

$$\text{Fluid velocity } V_w \text{ (cm/sec)} = \frac{q}{h_w} \tag{10}$$

$$\text{Time to Traverse strip } t\text{(sec)} = \frac{\Delta x}{V_w} \tag{11}$$

Gravity has t secs to act on a parcel of liquid passing through the section.

$$\text{Pressure gradient in oil} = \left(\frac{dP}{dD}\right)_{oil}$$

$$\text{Pressure gradient in water} = \left(\frac{dP}{dP}\right)_{water}$$

$$\text{Vertical Velocity } (V_v) = -\frac{k}{\mu_w}\left[\left(\frac{dP}{dD}\right)_{water} - \left(\frac{dP}{dD}\right)_{oil}\right] \tag{12}$$

$$\text{Vertical distance traveled in } (S_v) \quad \Delta x = V_v t \tag{13}$$

h_w for the next section can be estimated.

Finally:

$$h_w \text{ new} = h_w - S_v \tag{14}$$

Equipment and Procedure

A linear sand pack model has to be used to vindicate the formula derived in this paper. (See Illustration 1)

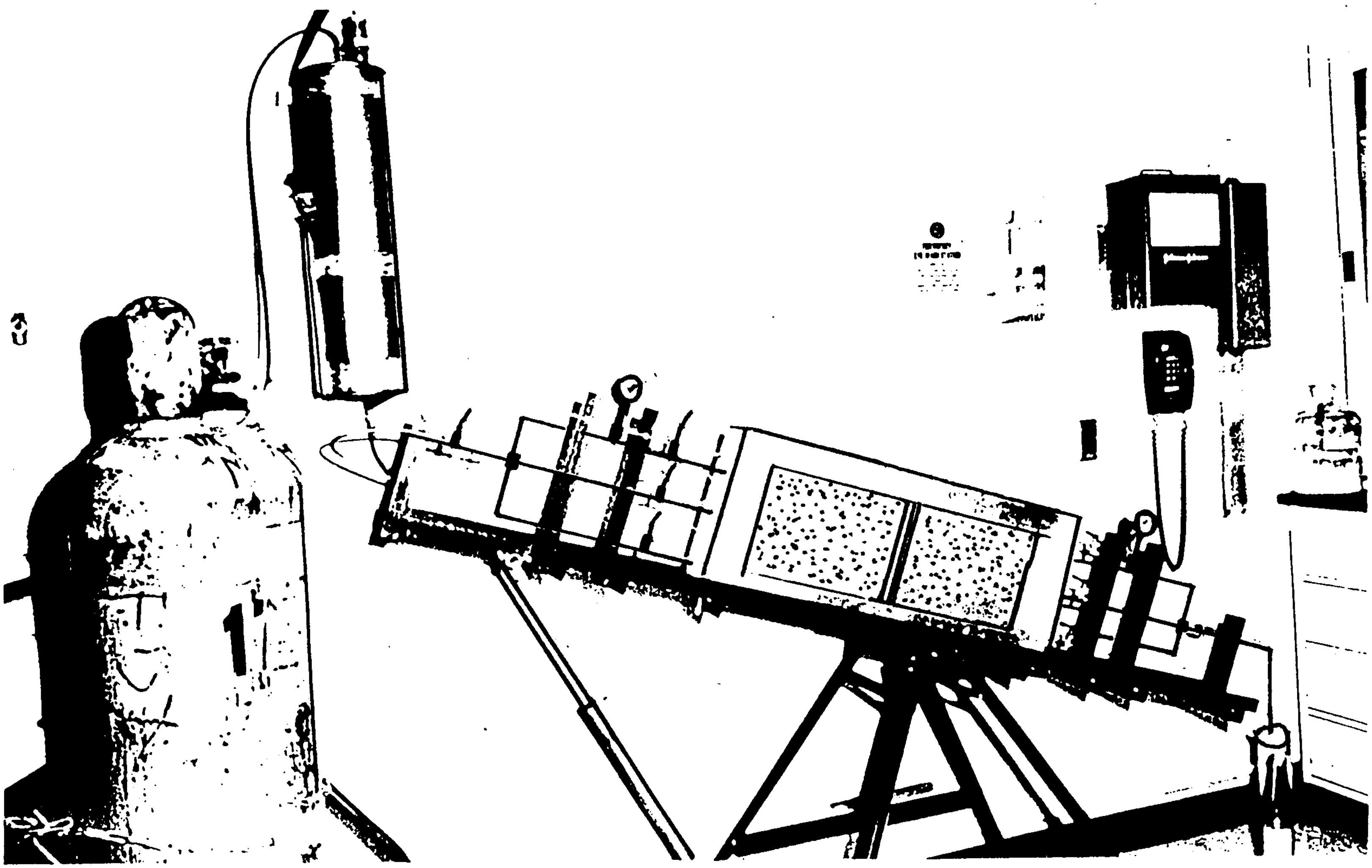


Illustration 1: Visual Displacement Model

The model is 21 cm heigh, 60 cm long and 1 cm in width. Sand is evenly sized (35-15 mech) and packed into the model using pneumatic vibrations over a twenty hour period. The result is a homogeneous sand pack.

The fluid drive mechanism is a constant rate peristaltic pump. Prior to the main flow experiments the pump is used to flow water (viscosity 1 cp) through the cell at constant rate to determine the sand permeability.[4] Porosity is found by fluid displacement and quartz density separately.[4]

$$\text{Porosity } (\Phi) \text{ of cell} = \frac{\text{Weight of sand in cell} \div \text{Density}}{\text{Bulk volume of cell density}} \qquad (15)$$

Fluid velocity in the cell must be laminar if it is to obey Darcy's Law. (Equation 1) To ensure this the Reynold's number[5] must be below 1. (Equation 16)

$$R_e = \frac{DV\rho_w}{\mu} \qquad (16)$$

The following is a list of steps for the actual flooding experiment.

1. Flood cell with mineral oil.
2. Set pump rate to ensure laminar flow.
3. Commence flooding with water (viscosified with glycerine: see scaling).
4. Maintain flood for at least five pore volumes.

Scaling

The laboratory model must be representative of the reservoir and certain parameters have to be scaled. To do this basic flow formulas have to be studied:[6]

$$V_o = - k \frac{k_o}{\mu_o} \frac{\partial P_o}{\partial x} \tag{17}$$

$$V_w = - k \frac{k_w}{\mu_w} \frac{\partial P_w}{2x} \tag{18}$$

Capillary pressure between the two fluids:

$$P_w = P_o + P_c \tag{19}$$

and

$$\frac{\partial P_w}{\partial x} = \frac{\partial P_o}{\partial x} + \frac{\partial P_c}{\partial x} \tag{20}$$

Apply the principle of mass conservation to each of the fluids.

$$\int \frac{\partial s}{\partial t} - \frac{\partial V_o}{\partial x} = 0 \text{ for the oil} \tag{21}$$

$$\int \frac{\partial s}{\partial t} + \frac{\partial V_w}{\partial x} = 0 \text{ for the water} \tag{22}$$

Equations 17, 18, 20, 21, and 22 form a system of five simultaneous equations defining the dependent variables. Upon elimination and further manipulation we can obtain basic dimensionless equations.

$$\frac{\partial S}{\partial T} + \frac{dF}{dS} \frac{\partial S}{\partial x} - \frac{k}{c} \cdot \frac{1}{LV\mu_w} \frac{\partial}{\partial x} [k_o \cdot F \cdot \frac{dP}{ds} \cdot \frac{\partial s}{\partial x}] = 0 \tag{23}$$

$$k_o[1 + \frac{k}{LV\mu_w} \cdot k_w \cdot \frac{dPc}{ds} \frac{\partial s}{\partial x}] = 0 \text{ at } x = 0 \text{ for any } T \tag{24}$$

where

$$X = \frac{x}{L} \tag{25}$$

$$T = \frac{tV}{L_t} \qquad (26)$$

Now analysis of Equation 23 and 24 show that the flood is also affected by the length of the system and rate of injection. Also the flood performance will be identical if the product LV_{μ_w} is the same in identical porous media.

$$\text{Scaling coefficient} = LV\mu_w \qquad (27)$$

In the flow experiments $LV\mu_w$ must be high enough to make capillary forces negligible as Equation 23 will reduce to:

$$\frac{dS}{dT} + \frac{dF}{dS}\frac{dS}{dx} = 0$$

Care should be taken to ensure that the scaling required to reduce capillary forces does not result in non-laminar flow.

Results and Discussions

To date the theory has been radically changed since its inception, in that the driving force has been included in the analytical technique and also a numerical technique has been developed.

The original scaling technique in the proposal has been changed and improved based on the literature review.

The laboratory model has been built. (Porosity - 31%, Permeability - 26 Darcy)

At the time of writing this report, initial scaling runs have been carried out in the model to ensure Laminar flow and homogenity of the sand pack. Initial packing of the model did not yield homogeneous sands.

Conclusion

Progress is being made to the vindication of the proposed water flood prediction techniques. The use of the model, properly scaled, will allow the formulas to be partially proven. It is hoped to find a suitable reservoir presently undergoing a water flood to give a real indication of the two techniques viability.

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Nomenclature

F = Dimensionless time function.

g = Acceleration due to gravity (cm/sec^2).

h = Total height of reservoir (cm).

he = Effective head and water due to imposed driving force (cm[1] H_2O)

h_o = Thickness of oil column (cm).

h_w = Thickness of water column (cm).

k = Permeability (Darcy).

L = Length of reservoir (cm).

Pc = Capillary pressure (atmosphere).

q_w = Water flow rate (cm^3/sec).

R_e = Reynolds number.

S = Dimensionless water saturation.

t = Time (secs).

T = Dimensionless time coordinate.

v = Velocity of fluid (cm/sec).

x = Relative distance.

ρ_w = Density of water (qm/cm^3)

ρ_o = Density of oil (qm/cm^3)

Φ = Hubbert's Potential.