Finite Element Glacier Dynamics Model Applied to Columbia Glacier, Alaska

By W. G. SIKONIA

STUDIES OF COLUMBIA GLACIER, ALASKA

GEOLOGICAL SURVEY PROFESSIONAL PAPER 1258-B

Description of a computer model indicating that Columbia Glacier will begin a rapid, catastrophic retreat in 1983

UNITED STATES GOVERNMENT PRINTING OFFICE, WASHINGTON:1982

UNITED STATES DEPARTMENT OF THE INTERIOR

JAMES G. WATT, *Secretary*

GEOLOGICAL SURVEY

Dallas L. Peck, *Director*

Library of Congress catalog-card No. 81-607563

For sale by the Superintendent of Documents, U.S. Government Printing Office
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SYMBOLS AND ABBREVIATIONS

A	Ice flow law parameter
a	Year
a	Calving law parameter
$\mathbf{a}^h$	Rate of ice-thickness-change vector
$\mathbf{a}^p$	Nodal pressure vector
$\mathbf{a}^u$	Nodal velocity vector
B	Constant in Budd sliding law
$\mathbf{B}$	Deformation rate shape function
b	Calving law parameter
$\dot{b}$	Annual ice balance
c	Calving law parameter
D	Discharge
$\mathbf{D}'$	Constitutive matrix
DOF	Degree of freedom
d	Day
d	Effective shear deformation rate
d_{ij}	Deformation rate tensor
$\mathbf{d}$	Deformation rate vector
δ	Indicates variation of following variable
∂	Used in partial derivative
$\mathbf{E}$	$\equiv (1/\mu)\, \mathbf{D}'$
$\mathbf{F(a)}$	Nonlinear ice-flow function of nodal variables $\mathbf{a}$
$\mathbf{f}$	Force vector
$\mathbf{G}$	Auxiliary matrix used in tangent matrix derivation
γ	Angle of surface
Γ_s	Ice surface
h	Total ice thickness; related quantities have this as superscript
h_b	Altitude of bed
h_s	Altitude of surface
h_u	Height of the ice column unsupported by water pressure
h_w	Water depth
$\overline{\mathbf{K}}$	Coefficient matrix related to spatial variation of flow field
$\mathbf{K}^h$	Coefficient matrix related to rate of ice thickness change
$\mathbf{K}, \mathbf{K}^u$	Coefficient matrix describing effect of ice viscosity
$\mathbf{K}^p$	Coefficient matrix related to pressure distribution
km	Kilometer
Mg	Megagram
$\mathbf{M}$	Mass matrix
m	Meter
μ	Viscosity
$\mathbf{N}$	Shape function matrix
n	Ice flow law parameter
Ω	Volume
p	Pressure; related quantities have this as superscript
r^2	Coefficient of determination
ρ_i	Density of ice
ρ_w	Density of water
s	Second
$\mathbf{s}$	Matrix used in determining emergence velocity
σ_{ij}'	Deviatoric stress tensor
$\boldsymbol{\sigma}'$	Deviatoric stress vector
T	Denotes transpose matrix as superscript
τ_b	Shear stress at the bed
UTM	Universal Transverse Mercator
u	Denotes velocity; related quantities have this as superscript
$\mathbf{u}$	Velocity vector
V_b	Sliding speed at the bed
V_c	Calving speed
V_e	Emergence velocity
V_s	Surface ice speed
$X(t)$	Position of calving terminus at time t

FINITE ELEMENT GLACIER DYNAMICS MODEL APPLIED TO COLUMBIA GLACIER, ALASKA

By W. G. Sikonia

ABSTRACT

A two-dimensional finite element computer model describes the dynamics of glacier flow and includes the flow of ice, sliding, calving, and surface balance. Surface profile and terminus position are followed in time. A calving law that relates calving speed to water discharge at the bed of the glacier and to the height of the ice column unsupported by water pressure is shown to fit observed calving at Columbia Glacier. Application of the flow model to Columbia Glacier indicates that catastrophic, rapid retreat from the terminal moraine will begin in 1983. This will produce an increase in the production of icebergs that can drift into oil tanker lanes to and from Valdez, Alaska.

INTRODUCTION

Columbia Glacier, Alaska (fig. 1), is in a state precariously close to catastrophic retreat (Post, 1975; Meier and others, 1978; Sikonia and Post, 1980; Meier and others, 1980a, b). A dynamic model of glacier behavior was sought that could adequately represent at least the lowest 14 km of the glacier; the model must be capable of dealing with a large amount of sliding, with flow up a terminal moraine, and with a moving calving face. Bindschadler applied a one-dimensional model (Bindschadler, 1978; Meier and others, 1980b), but it seems questionable whether the approximations of the model adequately represent reality near the calving face, especially with regard to surface-slope averaging schemes and longitudinal stresses.

THE FINITE ELEMENT MODEL

The two-dimensional finite element model described in this paper is an attempt to relate more adequately the motion of the ice mass to the physical processes involved. However, the two-dimensional nature of the model dictates that it also is an idealization of reality. The model represents a vertical cross section through a glacier along a central flow line, but no attempt has been made yet to include effects of lateral shear stresses or lateral deformation rates.

The ice flow law describing the deformational contribution to ice motion is

$$d_{ij} = A\tau^{n-1}\sigma'_{ij}, \qquad (1)$$

where d_{ij} is the deformation rate tensor; σ'_{ij}, the deviatoric stress tensor; τ, the effective shear stress defined by

$$\tau = (\tfrac{1}{2}\sigma'_{ij}\sigma'_{ij})^{\frac{1}{2}} \qquad (2)$$

(summation convention on subscripts i,j); and A and n are constants, with approximate values of

$$A = 0.140\ \mathrm{bar}^{-n}a^{-1} \text{ and } n = 3. \qquad (3)$$

Sliding is modeled as a layer of very soft ice (large valued A) along the bed. In the Columbia Glacier application, the layer is assumed to be a constant 10 m thick, with $n=3$ and A ranging from 7 to 40 $\mathrm{bar}^{-n}\mathrm{a}^{-1}$, fitted along the bed so that the correct sliding is obtained to match measured surface velocities. This is approximately equivalent to use of a sliding law suggested by Budd and others (1979) in the form

$$V_b = B\tau_b^{\,n}/h_u. \qquad (4)$$

Here V_b is the sliding speed on the bed; B, a constant perhaps approximately 1,800 $\mathrm{m^2a^{-1}bar^{-n}}$ for $n=3$; τ_b, the shear stress at the bed; and h_u, the ice-column height unsupported by water pressure

$$h_u = h - h_w\rho_w/\rho_i, \qquad (5)$$

where h is total ice depth; h_w, water depth; and ρ_w, ρ_i, the densities of water and ice, respectively. Bindschadler (written commun., 1978) found that this sliding law fits Variegated Glacier, Alaska, data.

The solution proceeds in a sequence of time steps. At each step, equilibrium iterations using Newton's method are done first to produce consistent stress-velocity fields described by the nonlinear equation 1. By using the velocity field, the surface and calving front then are projected to new positions, and the process stepped in time.

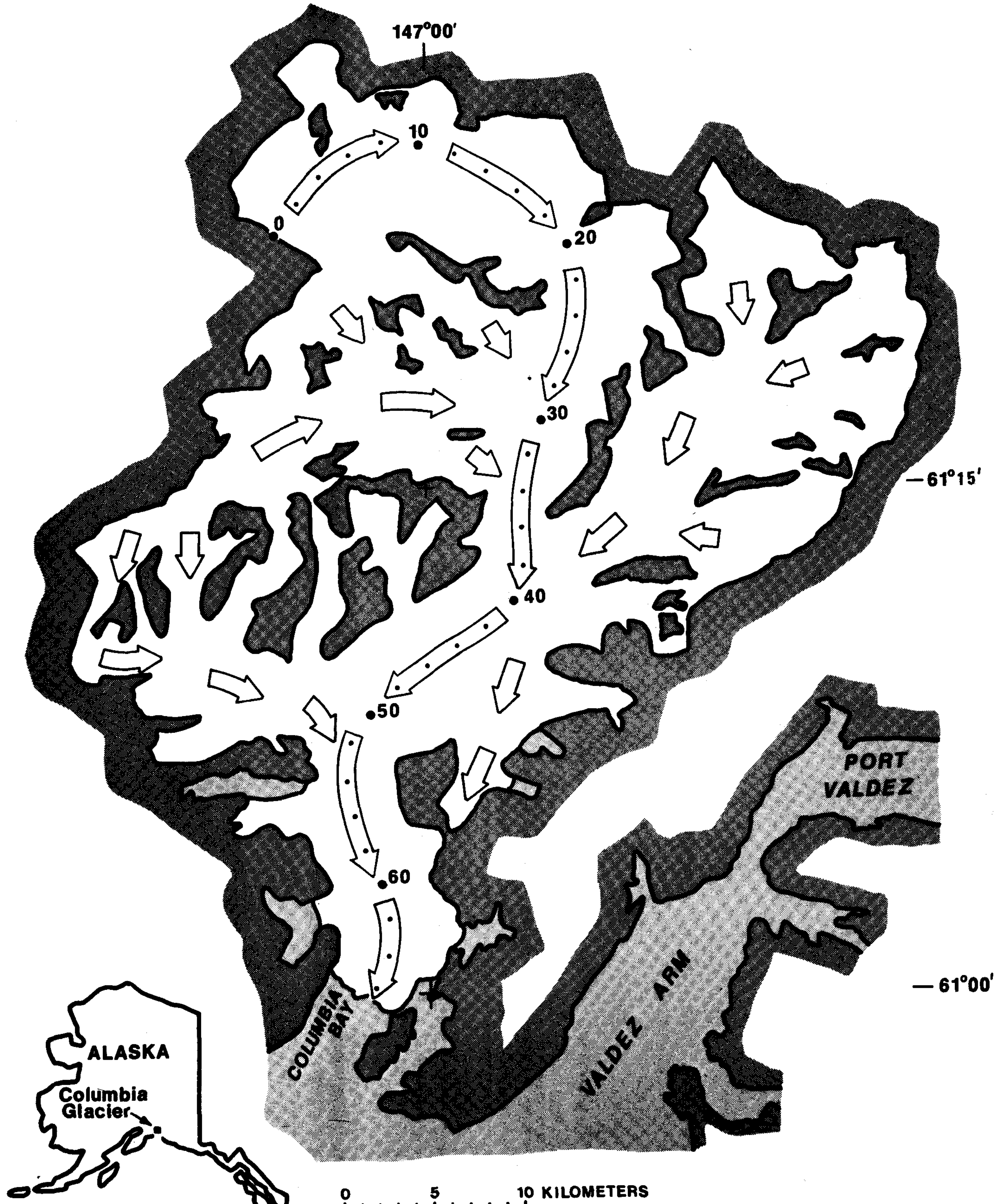


FIGURE 1.–Index map of Columbia Glacier, Alaska. Arrows show direction of flow. Main ice stream is indicated by longer arrows and dots at 2-km intervals along the longitudinal coordinate system.

The finite element framework for the computer program is a general purpose program (Taylor, 1980; Zienkiewicz, 1977, p. 733). Velocity and pressure are chosen as the variables, and the treatment follows that of Zienkiewicz (1977, p. 612) except that, for ice, the viscosity, μ, is not constant but is rather a function of the effective shear stress,

$$\mu = 1/(2A\tau^{n-1}). \quad (6)$$

Also, kinetic effects can be ignored because of the slow creep of ice. Because the viscosity is nonconstant, the equations are nonlinear, and the tangent stiffness matrix for use in Newton-Raphson iteration takes the appropriate derivatives of viscosity into account. Other finite element programs with a similar approach are attributable to Schmidt (1977), Raymond (1978), and Hooke and others (1980), but these treat only geometrically static, purely creeping flow. The program described in this paper treats deformational motion, sliding, balance, and calving effects in a geometrically dynamic setting. It also starts with a good, readily available general purpose finite element framework; the advantage of this approach is that only the specific program modules directly concerned with the glacier dynamics need be considered, greatly simplifying programing, documentation, and orientation of new users.

To take into account the dynamics, the movement of the upper free surface must be followed. This surface is followed essentially as in Zienkiewicz (1977, p. 623); the free surface must be projected along the surface velocity vectors to its new position. The program makes this projection by defining an auxiliary variable – the vertical surface speed – decomposing the surface velocity into general nonorthogonal components along the surface and in the vertical direction. In addition to the movement of the upper free surface, the movement of the calving face is projected from one time step to the next by using the calving law described in the section on "Calving Law." Because of large changes in the configuration of the ice mass, the entire mesh must be reconstructed at each time step.

CALVING LAW

The position of the calving terminus, $X(t)$ (assumed vertical), in time is determined by the imbalance between the horizontal surface ice speed, V_s, at the terminus (positive downglacier) and the calving speed, V_c (positive upglacier),

$$X = V_s - V_c. \quad (7)$$

Alternatively, equation 7 defines calving speed, V_c, as the rate of retreat of the calving terminus in the moving ice stream. A calving law relating calving speed linearly to water depth has been suggested (Meier and others, 1980b). The suggested water-depth law was restricted to yearly averaged data that was averaged also across the width of the glacier. It definitely does not apply for intervals of less than a year for calving at a single location on the terminus. When applied to the calving of Columbia Glacier at the head of the embayment, by using photogrammetrically determined terminus positions taken at approximately 2-month intervals from July 24, 1976, to September 2, 1980, the water-depth law yields a coefficient of determination, r^2, of only 0.08. However, for Columbia Glacier, calving speed appears to be related to water discharge at the bed (Sikonia and Post, 1980). A law that does appear to fit the observed intrayearly calving at Columbia Glacier is (see also fig. 2 and table 1)

$$V_c = aD^b h_w^c. \quad (8)$$

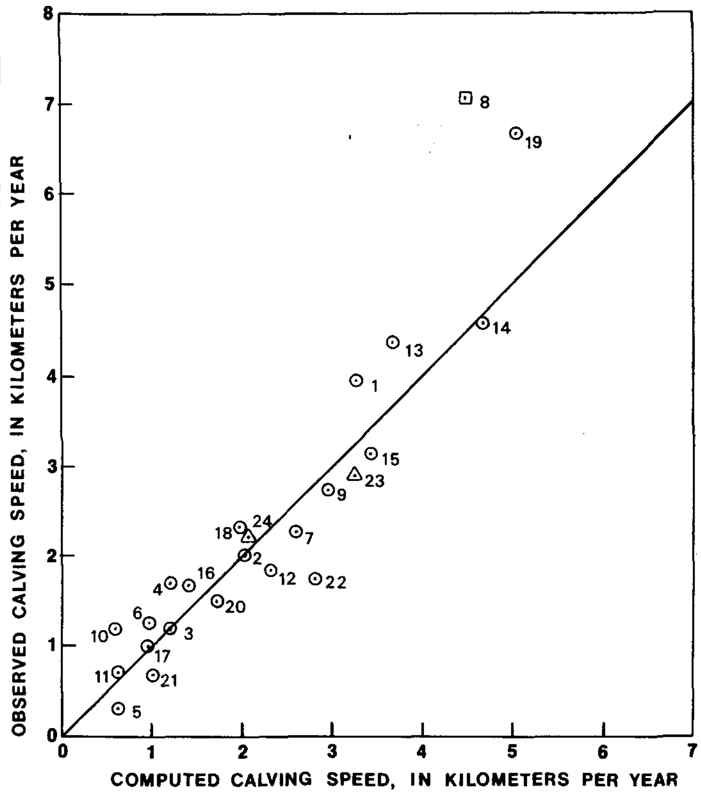


FIGURE 2.—Calving law $V_c = aD^b h_w^c$, where $a = 1.092\times10^3$ (km/a) $(m^3/s)^{-b} m^{-c}$, $b = 0.5689$, and $c = -2.175$. V_c is speed of calving at the head of the embayment, D is the discharge of the Knik River, and h_w is the height of the unsupported ice column. The data points, as given in table 1, were determined from analysis of aerial photographs. Point 8 corresponds to a time interval during which Kadin Lake suddenly released a large volume of water into the subglacial flow; in this interval, the Knik River discharge would not correlate well with this flow. Points 8, 23, and 24 were not used in the regression that fitted the constants a, b, and c.

TABLE 1.—*Calving law data*

[N, data interval number. t_1 and t_2, beginning and ending dates of interval. Δt, number of days in the interval. h_{g1} and h_{g2}, ice-cliff heights at head of embayment at times t_1 and t_2, respectively. h_{w1} and h_{w2}, water depths at head of embayment at times t_1 and t_2, respectively. D, average discharge of Knik River during interval. h_u, average unsupported height during interval. $(V_c)_o$ and $(V_c)_c$, observed and calculated calving speeds at head of embayment. ΔV_c, difference between calculated and observed calving speed. Calculated speed is based on the calving law $V_c = aD^b h_u^c$]

N	t_1	t_2	Δt (d)	h_{g1} (m)	h_{g2} (m)	h_{w1} (m)	h_{w2} (m)	D (m³/s)	h_u (m)	$(V_c)_o$ (m/a)	$(V_c)_c$ (m/a)	ΔV_c (m/a)
1	07/24/76	10/01/76	69	98	86	127	180	464.65	71.87	3,964	3,293	-671
2	10/01/76	11/17/76	47	86	86	180	175	119.82	62.73	2,008	2,047	39
3	11/17/76	01/19/77	63	86	88	163	146	59.96	66.74	1,190	1,207	17
4	01/19/77	03/07/77	47	85	89	155	142	63.45	67.53	1,697	1,215	-482
5	03/07/77	04/23/77	47	88	94	137	100	30.68	75.46	307	631	324
6	04/23/77	06/02/77	40	94	100	100	80	110.07	85.20	1,259	1,002	-257
7	06/02/77	07/07/77	35	99	95	80	109	486.96	84.61	2,256	2,371	115
8	07/07/77	08/29/77	53	93	99	102	278	765.94	71.09	7,059	4,481	-2,578
9	08/29/77	11/08/77	71	100	78	200	200	231.72	62.78	2,738	2,974	236
10	11/08/77	02/28/78	112	81	97	170	125	21.12	69.66	1,175	607	-568
11	02/28/77	04/19/78	50	96	84	125	112	30.90	74.46	712	652	-60
12	04/19/78	06/11/78	53	68	81	110	113	126.76	59.88	1,832	2,339	509
13	06/11/78	07/30/78	49	80	94	112	177	460.50	68.05	4,362	3,689	-673
14	07/30/78	08/26/78	27	90	90	160	182	684.48	67.58	4,588	4,692	104
15	08/26/78	11/08/78	74	92	77	185	210	230.27	58.61	3,150	3,442	292
16	11/08/78	01/06/79	59	72	76	215	182	22.46	47.97	1,657	1,415	-242
17	01/06/79	04/12/79	96	72	77	182	140	17.36	53.39	1,004	969	-35
18	04/12/79	08/18/79	128	93	111	85	135	412.75	87.58	2,318	2,002	-316
19	08/18/79	10/20/79	63	70	76	82	195	350.34	54.84	6,669	5,049	-1,620
20	10/20/79	02/29/80	132	77	75	195	167	44.71	52.27	1,478	1,737	259
21	02/29/80	05/12/80	73	74	90	167	130	35.61	62.53	683	1,034	351
22	05/12/80	07/22/80	71	80	101	110	137	390.94	74.31	1,748	2,775	1,027
23	07/22/80	09/02/80	42	92	89	90	118	587.54	76.86	2,880	3,252	372
24	09/02/80	10/30/80	58	91	85	110	133	207.33	72.07	2,205	2,068	-137

V_c is the calving speed at the head of the embayment, and h_u is the ice column height unsupported by water pressure, as defined in equation 5. The variable D would most appropriately be the actual discharge at the bed of the Columbia Glacier, but because actual discharge would be almost impossible to measure, a related discharge, that of nearby Knik River, was used instead (see Sikonia and Post, 1980). Constants a, b, and c have fitted values as follows:

$$a = 1.092 \times 10^6 \text{ (m/a)(m}^3\text{/s)}^{-b}\text{m}^{-c}, \quad b = 0.5689, \quad c = -2.175, \tag{9}$$

$$\rho_w = 1.018 \text{ Mg/m}^3, \quad \rho_i = 0.900 \text{ Mg/m}^3. \tag{10}$$

The standard error for the Columbia data is 529 m/a (in calving speeds ranging from 307 to 4,588 m/a), with a coefficient of determination of the logarithmically derived corresponding linear regression problem of $r^2 = 0.83$. (Although 1.018 Mg/m³ is the measured density of seawater in front of Columbia Glacier, the modeling computer runs (see section entitled "Modeling Columbia Glacier") actually were made by using $\rho_w = 1.000$ Mg/m³ and $\rho_i = 0.900$ Mg/m³. In this case, $a = 1.798 \times 10^6$ (m/a)(m³/s)$^{-b}$m^{-c}, $b = 0.5510$, $c = -2.239$, $r^2 = 0.83$, and the standard error was 369 m/a, based on fewer data points than the constants in equation 9 because of limited data availability at the time. It was not considered worth rerunning the program for these minor changes in constants.)

It is uncertain exactly what the relation between the discharge-unsupported height calving law and the water-depth law is, but there appear to be at least three possibilities:

1. One or the other law is wrong.
2. Some, as yet, not understood process averages the above law into the water-depth law for time periods of a year or longer.
3. The two laws are describing two, as yet, undistinguished distinct calving processes.

The discharge-unsupported height law is used in this study for reasons discussed in this section. This law also accommodates the observation that embayments form at the location of the main water discharge from the terminus and supports the physical idea that the closer the terminus is to floating, the more easily this discharge can bring about calving and ice removal from the face.

MODELING COLUMBIA GLACIER

The finite element model was applied to the lowest 14 km of Columbia Glacier to gain a better understanding of its possible future behavior (see also figs. 3 and 4 and table 2). The constant a in the calving law is too large for an idealized two-dimensional glacier; it produces an immediate catastrophic retreat, from the starting modeling date 1978.150, that has not occurred on the real glacier. This is understandable because embayments and related surface lowering are filled partly by flow from the sides, a factor which the two-dimensional model does not take into account. Also there is an "arch-bridging" stress effect within the ice around an embayment that could partly support the ice cliff at the head of the embayment, thus providing a larger unsupported height, h_u, and a smaller calving speed, V_c. This effect also is not taken into account in the two-dimensional model. An a corresponding to width-averaged calving is needed. Rather than trying to derive the averaging effect, the exponents b and c were fixed, and the a parameter adjusted for the two-dimensional glacier to produce future behavior that seems reasonable in view of the actual configuration of the glacier from early 1978 to the present.

Future behavior is extremely sensitive to changes in the a parameter. For example, $a=1168750$ produced a collapse that could not be followed with a time step of 0.025 year by 1982.075; the terminus had retreated 2,285 m by 1982.025. Decreasing a by only 0.27 percent to $a=1165625$ extended extremely rapid collapse to 1984.600, the terminus retreating 3,031 m by 1984.525. Decreasing a by another 0.27 percent to $a=1162500$ produced a gradual advance to the crest of the terminal moraine by 1982.400 (see also figs. 5, 6, 7, 8, and 9). The modeling also showed that a collapse is preceded by thinning of the glacier to the point that the unsupported height factor, h_u, of the calving law is small enough to produce a large calving speed when applied in equation 8; for example, $h_u=7$ m at the terminus, time 1982.250 and $a=1168750$.

The sensitivity of the glacier's behavior to the a parameter may be overstated or at least compressed in time scale because of the two-dimensional idealization, but, nevertheless, the behavior of the real glacier is probably still quite dependent on relatively small changes in the calving. As indicated by the calving law, for the real glacier, decreased calving that would stall collapse could be brought about by low runoff or by outlet of the major discharge at a location having a relatively high ice cliff and shallow water depth. Reference to figure 7 shows that h_u was relatively large in 1980, and, as indicated on figure 6, the retreat was, indeed, slowed that year. However, heavy rains in July and August 1981, together with a partial outburst of Kadin Lake, produced a large discharge that caused a large embayment to form. The data presented in figure 6 show that the observed terminus position is following again the retreat predicted by the model using $a=1165625$ and that the indicated collapse of Columbia Glacier is imminent.

TABLE 2.—*Flow-line data*

[Col., column number. Easting and Northing, UTM coordinates. h_s, ice surface altitude. h_b, bed altitude. $\dot{b}$, annual balance. V_s, surface speed. $\dot{h}$, annual surface altitude change]

Col.	Easting (m)	Northing (m)	h_s (m)	h_b (m)	$\dot{b}$ (m/a)	V_s (m/a)	$\dot{h}$ (m/a)
1.00	496047.77	6776061.73	465	−278	−2.67	950	−2.6
2.00	496190.00	6775312.92	430	−247	−3.42	1,030	−3.8
3.00	496212.11	6774551.05	411	−262	−3.98	960	−4.0
4.00	496069.20	6773802.37	401	−238	−4.45	922	−4.7
5.00	495989.61	6773044.34	370	−204	−4.90	935	−3.2
6.00	496058.28	6772285.25	340	−191	−5.41	930	−2.8
7.00	496296.14	6771561.12	314	−224	−5.98	900	−1.9
8.00	496606.30	6770864.89	290	−320	−6.30	839	−6.3
9.00	496867.26	6770148.76	275	−382	−6.32	808	−8.6
10.00	497094.55	6769421.24	264	−395	−6.29	812	−6.0
11.00	497283.81	6768682.92	248	−384	−6.26	866	−6.9
12.00	497449.85	6767939.03	226	−383	−6.27	904	−7.3
13.00	497519.90	6767180.06	210	−402	−6.36	933	−6.9
14.00	497522.32	6766417.87	198	−401	−6.53	1,012	−8.5
15.00	497459.13	6765657.99	178	−402	−6.68	1,102	−9.6
15.50	497402.49	6765281.07	167	−402	−6.76	1,150	−9.5
16.00	497340.59	6764905.06	159	−382	−6.91	1,210	−9.0
16.50	497273.44	6764529.95	152	−355	−7.09	1,320	−8.4
17.00	497201.03	6764155.75	140	−320	−7.21	1,460	−8.2
17.50	497115.89	6763784.20	129	−286	−7.38		−7.3
18.00	497024.00	6763414.39	118	−260	−7.63		−3.0
18.25	496975.53	6763230.14	113	−242	−7.77		−0.7
18.50	496925.37	6763046.32	108	−221	−7.87		1.0
18.75	496873.53	6762862.94	100	−196	−7.93		1.1
19.00	496820.00	6762680.00	96	−162	−7.94		0.1
19.25	496759.80	6762499.20	96	−135	−7.94		−1.1
19.50	496699.05	6762318.60	96	−106	−7.94		−2.5
19.75	496637.76	6762138.18	96	− 60	−7.94		
20.00	496575.92	6761957.94	96	− 10	−7.94		
20.25	496513.54	6761777.91		− 68			
20.50	496450.61	6761598.05		−132			
20.75	496387.14	6761418.37		−162			
21.00	496323.12	6761238.89		−168			
21.25	496257.12	6761060.15		−178			
21.50	496190.01	6760881.82		−182			
21.75	496121.77	6760703.90		−193			
22.00	496052.42	6760526.38		−202			

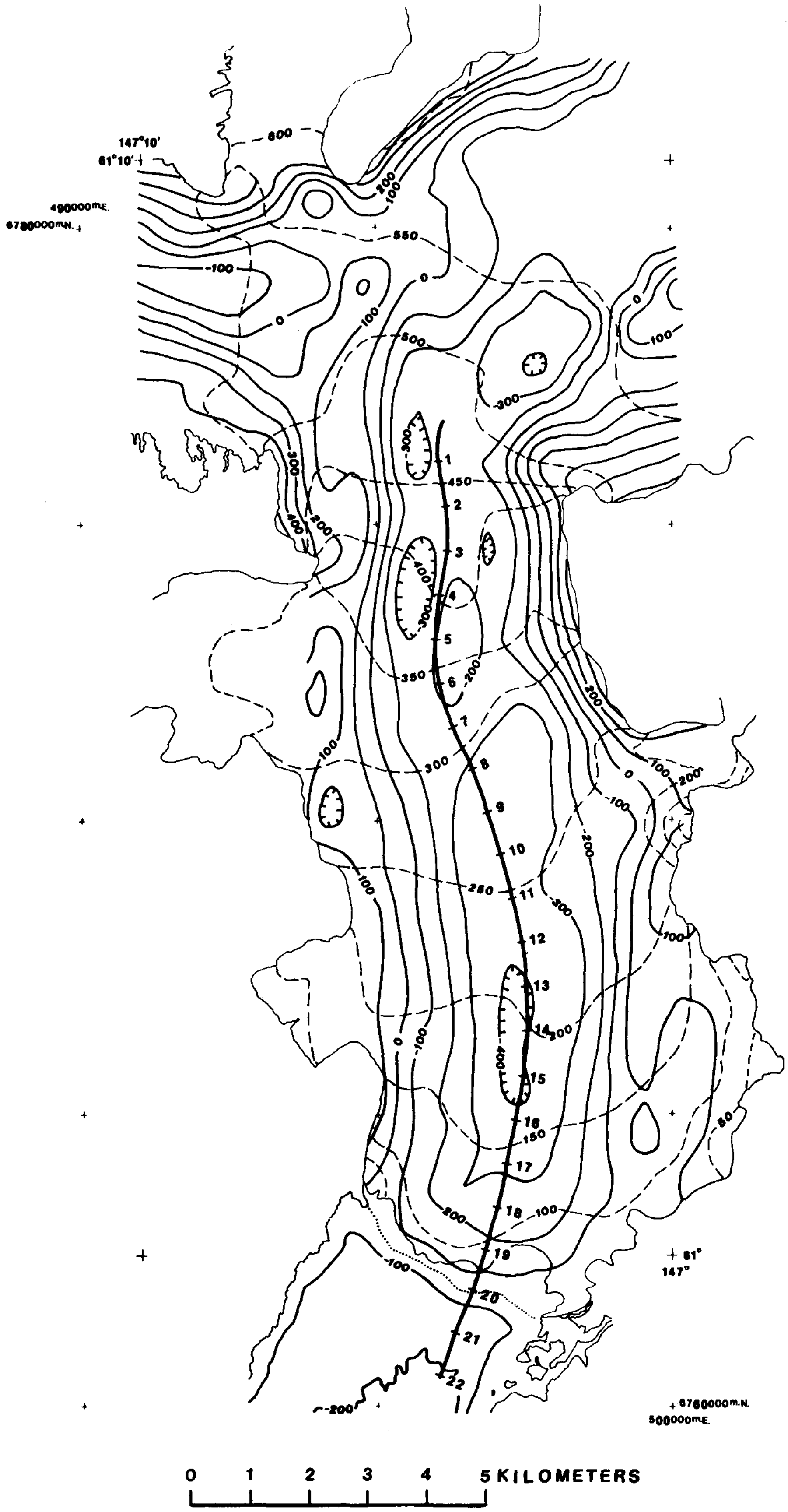


FIGURE 3. – Flow line used in the modeling. The numbers along the flow line are the column numbers, and UTM locations and data correspond to those given in table 2. Bed contours are solid, surface contours are dashed, and the terminal moraine is dotted (column locations 21 and 22 are in Columbia Bay). Kadin Lake is at UTM coordinates 490000 m.E., 6774000 m.N.

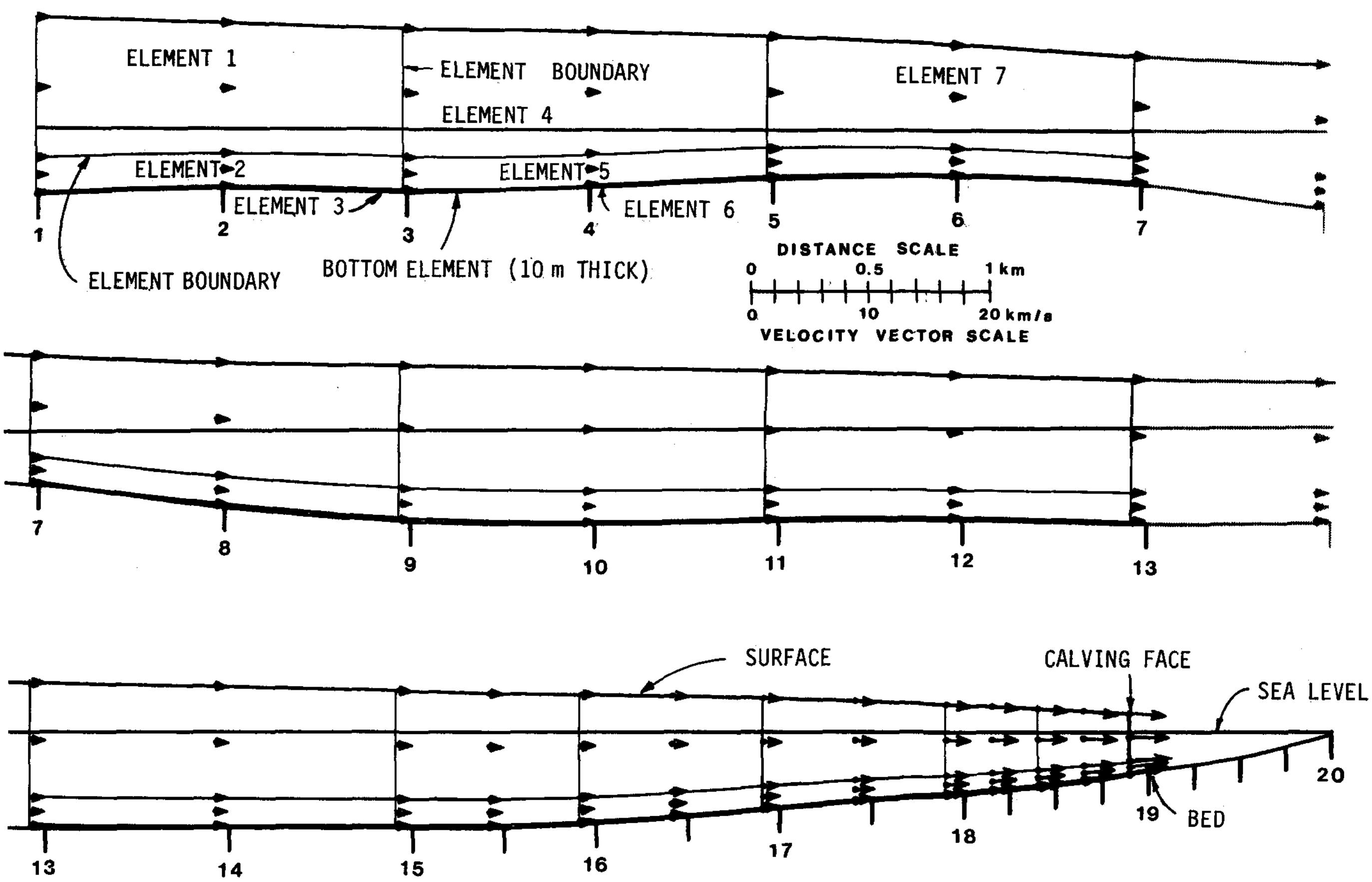


FIGURE 4.—Finite element grid. There are three elements through the thickness, with the bottom element a uniform 10 m in thickness. The ice velocity vectors are scaled to represent 0.05 year of displacement (if the ice continued beyond the velocity sampling points in straight lines, which, in general, it does not do).

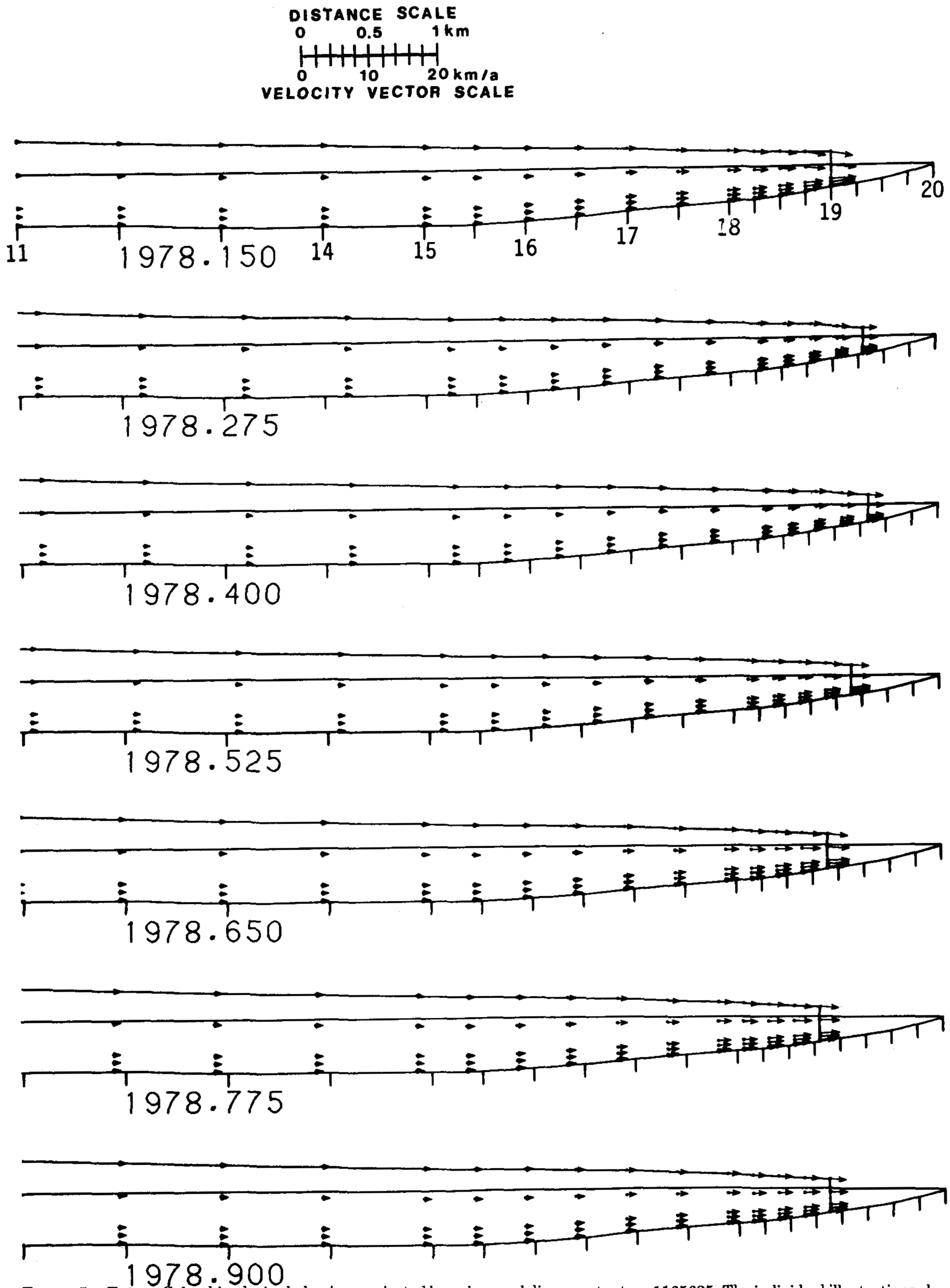


FIGURE 5.–Future Columbia glacier behavior, projected by using modeling constant $a = 1165625$. The individual illustrations show the flow-line profile from column locations 11 through 20 as a function of time.

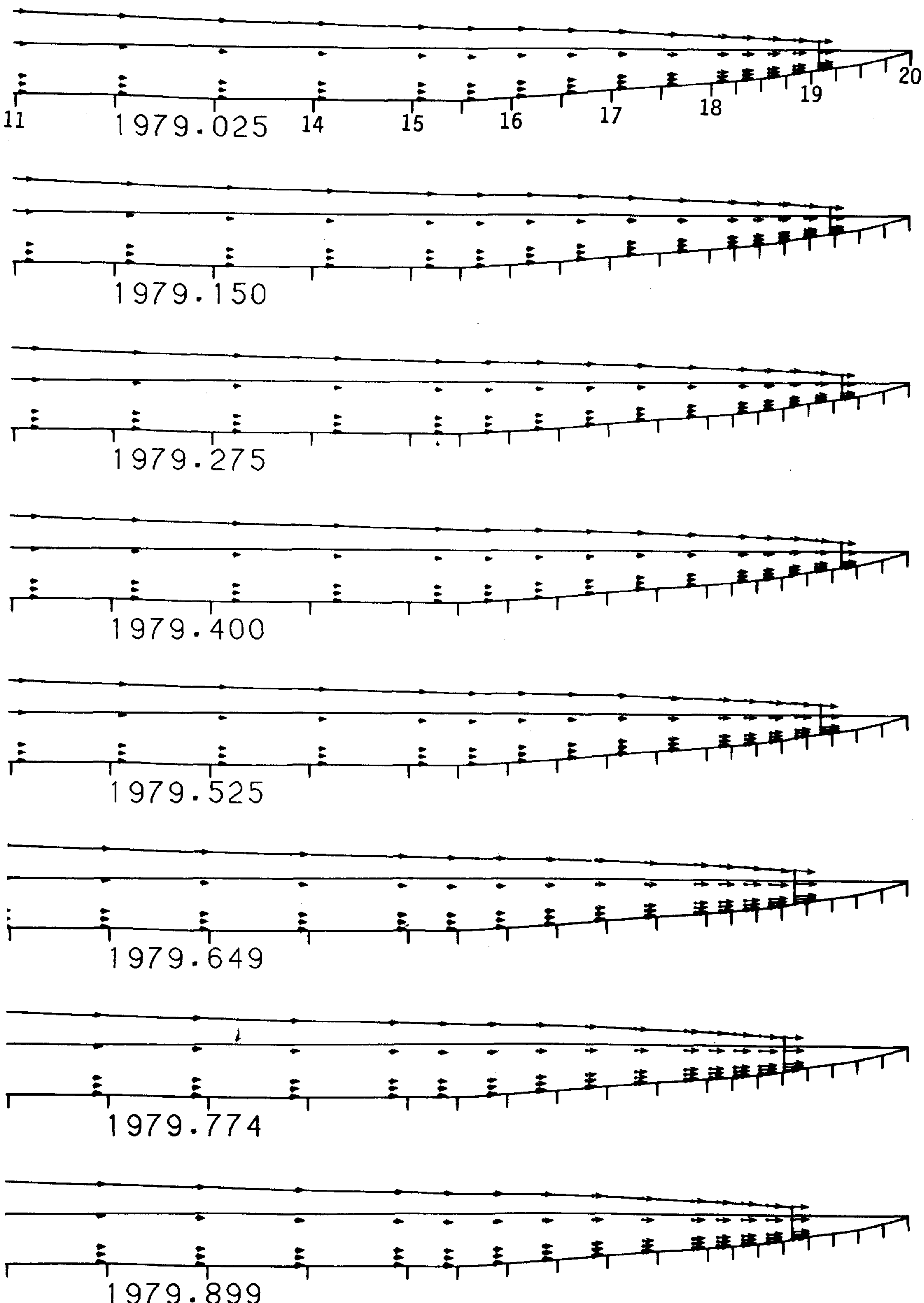
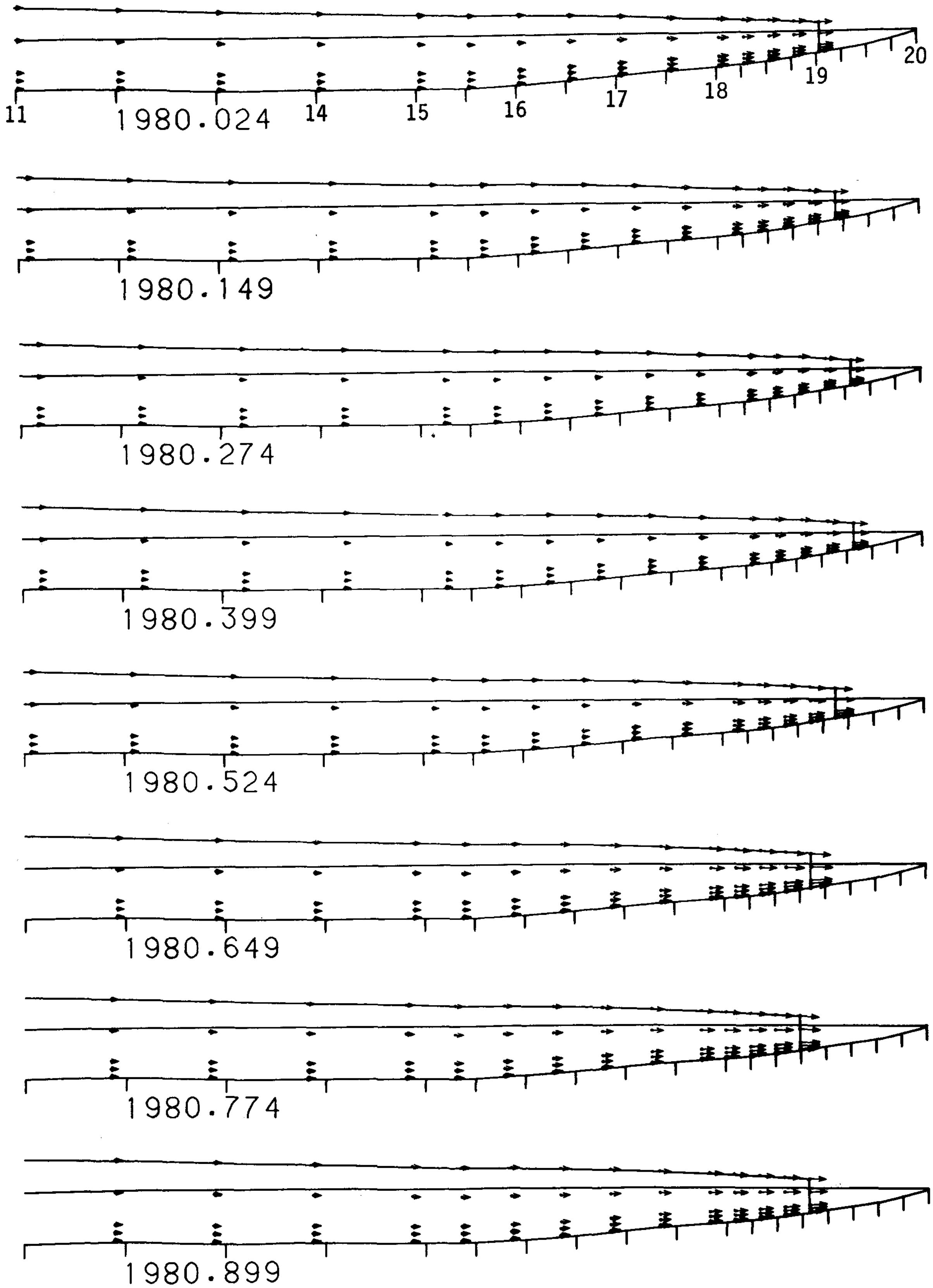


FIGURE 5 (Continued). – Future Columbia glacier behavior, projected by using modeling constant $a = 1165625$. The individual illustrations show the flow-line profile from column locations 11 through 20 as a function of time.



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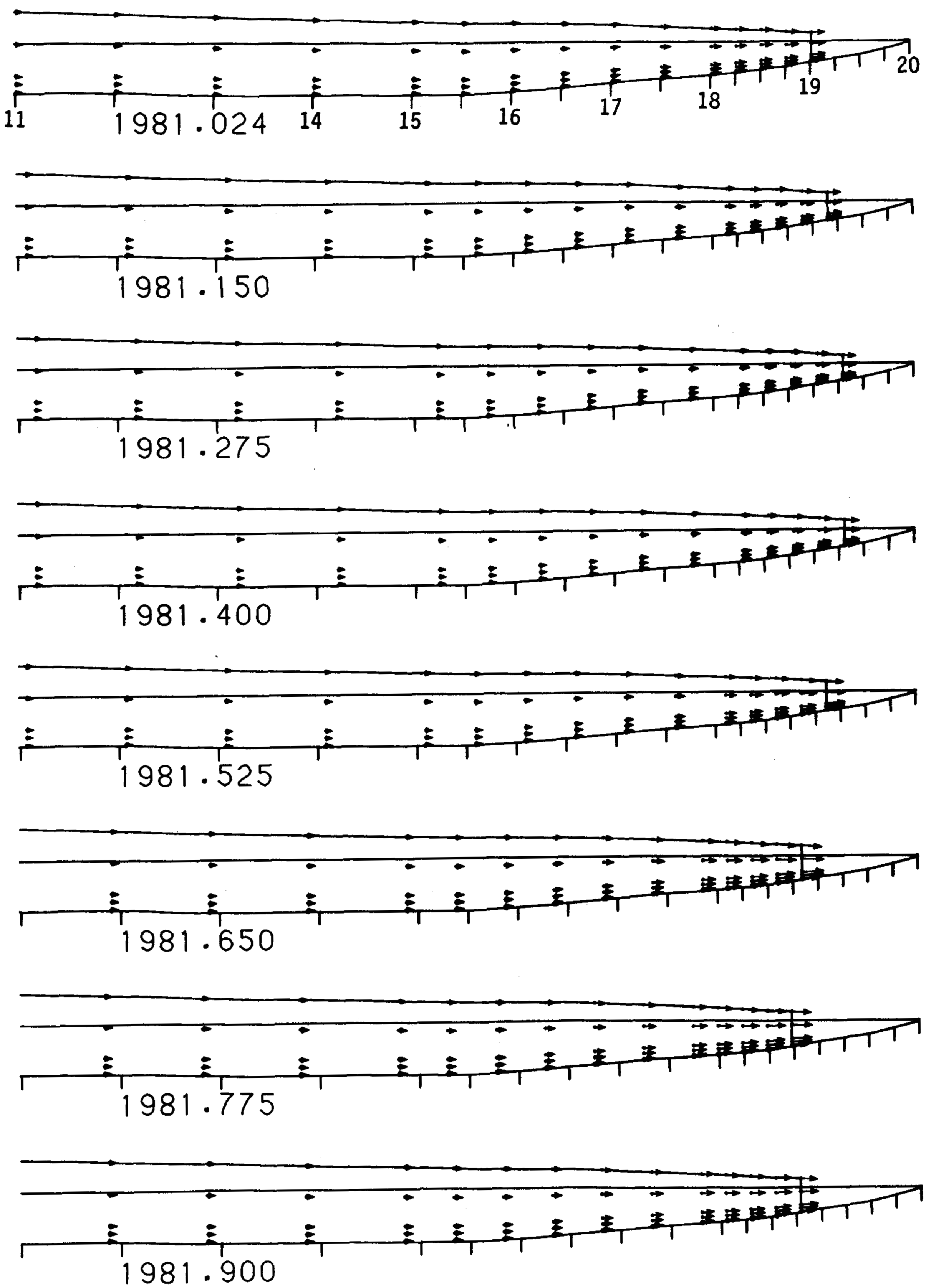


FIGURE 5 (Continued).–Future Columbia glacier behavior, projected by using modeling constant $a=1165625$. The individual illustrations show the flow-line profile from column locations 11 through 20 as a function of time.

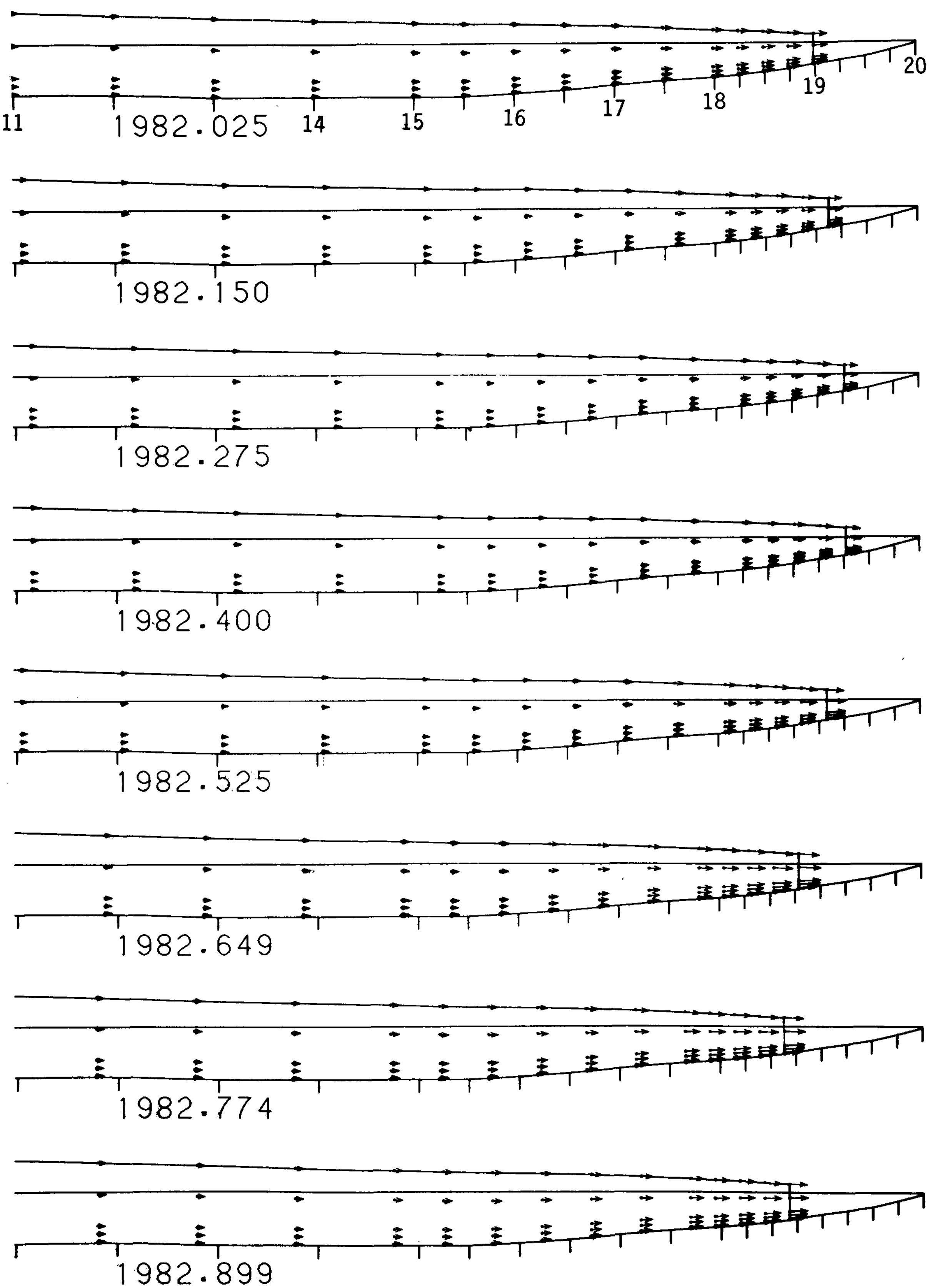


FIGURE 5 (Continued).–Future Columbia glacier behavior, projected by using modeling constant a=1165625. The individual illustrations show the flow-line profile from column locations 11 through 20 as a function of time.

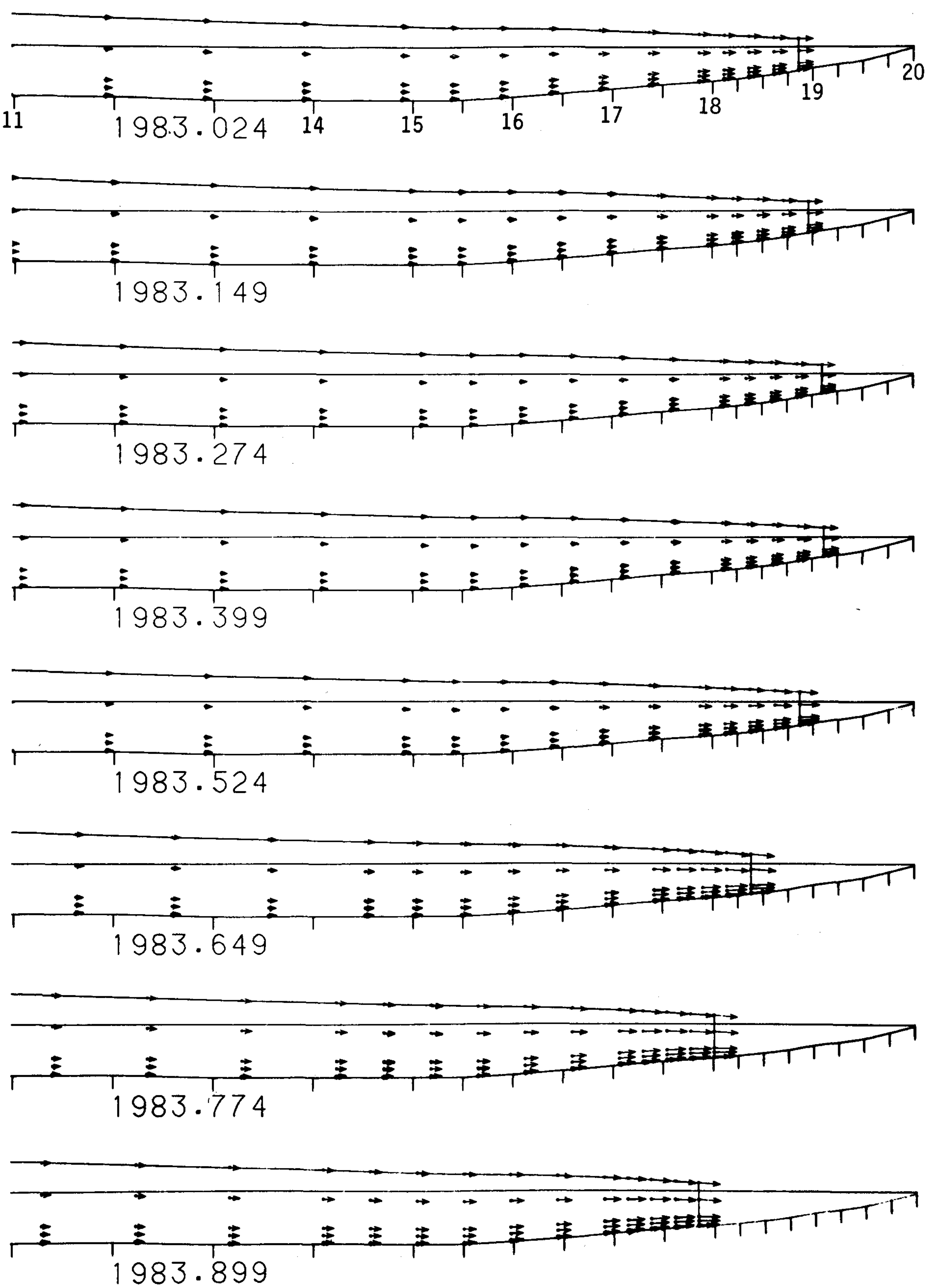


FIGURE 5 (Continued).—Future Columbia glacier behavior, projected by using modeling constant $a = 1165625$. The individual illustrations show the flow-line profile from column locations 11 through 20 as a function of time.

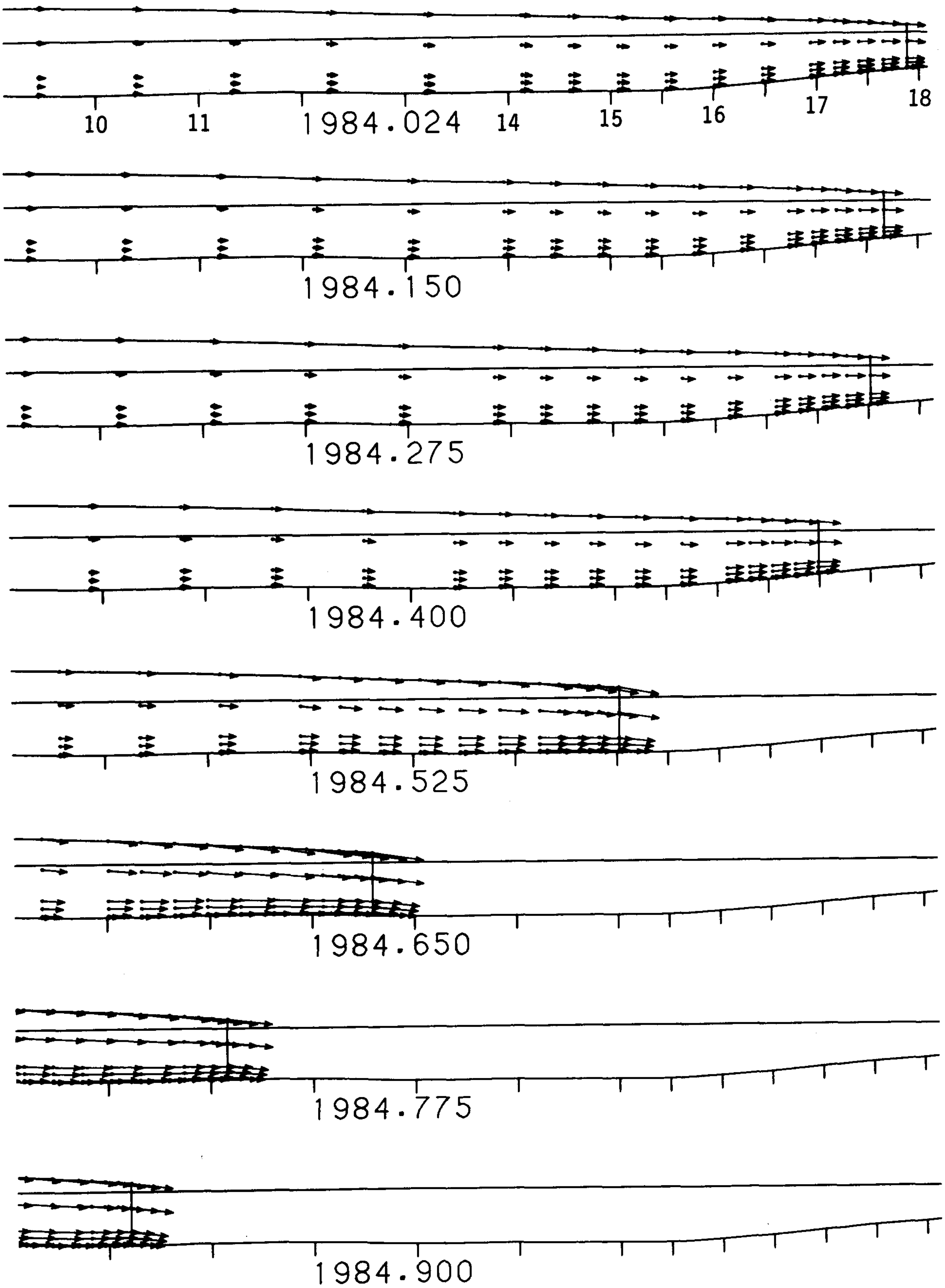


FIGURE 5 (Continued). – Future Columbia glacier behavior, projected by using modeling constant $a = 1165625$. The individual illustrations show the flow-line profile from column locations 10 through 18 as a function of time.

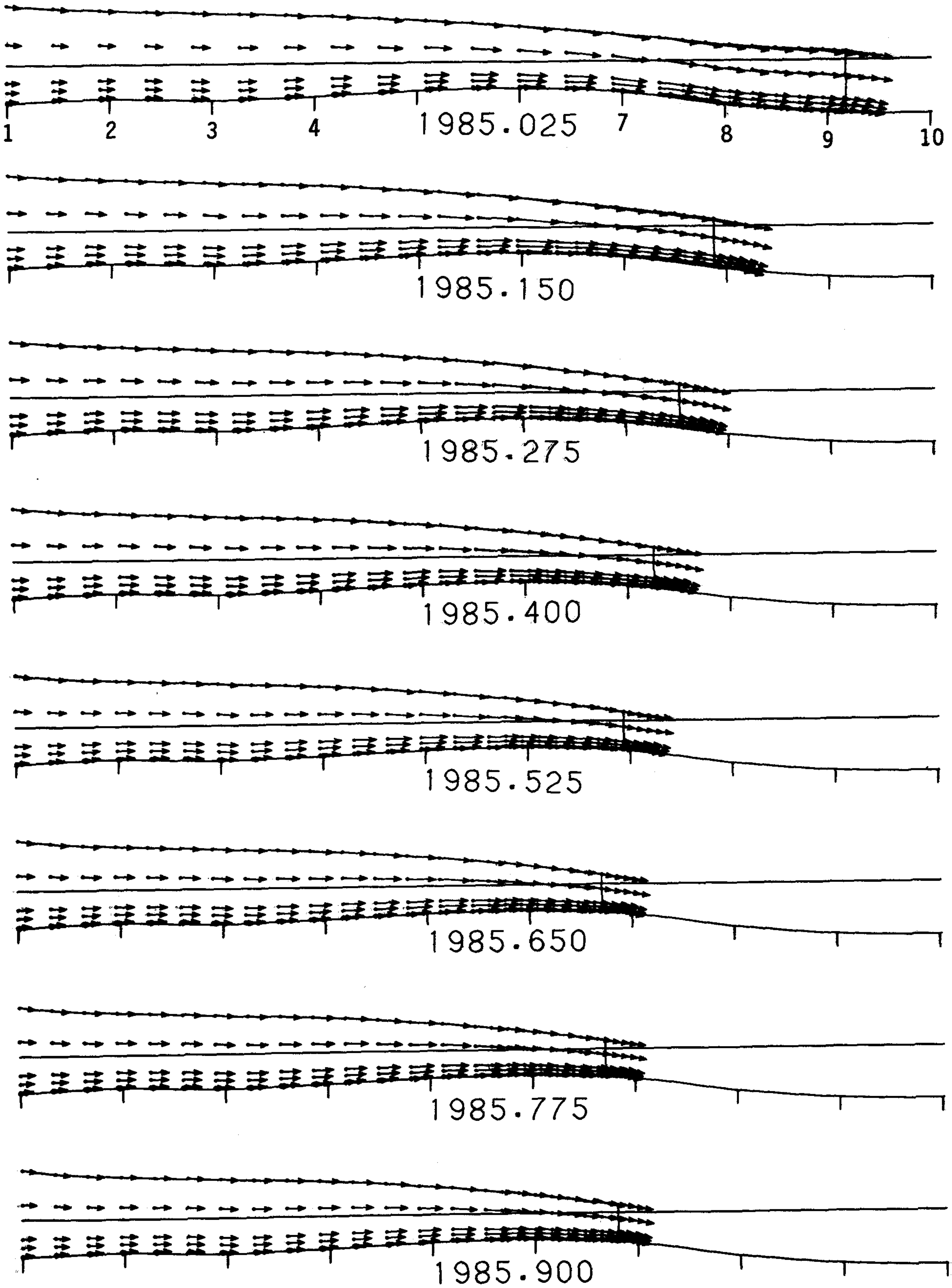


FIGURE 5 (Continued).–Future Columbia glacier behavior, projected by using modeling constant $a=1165625$. The individual illustrations show the flow-line profile from column locations 1 through 10 as a function of time.

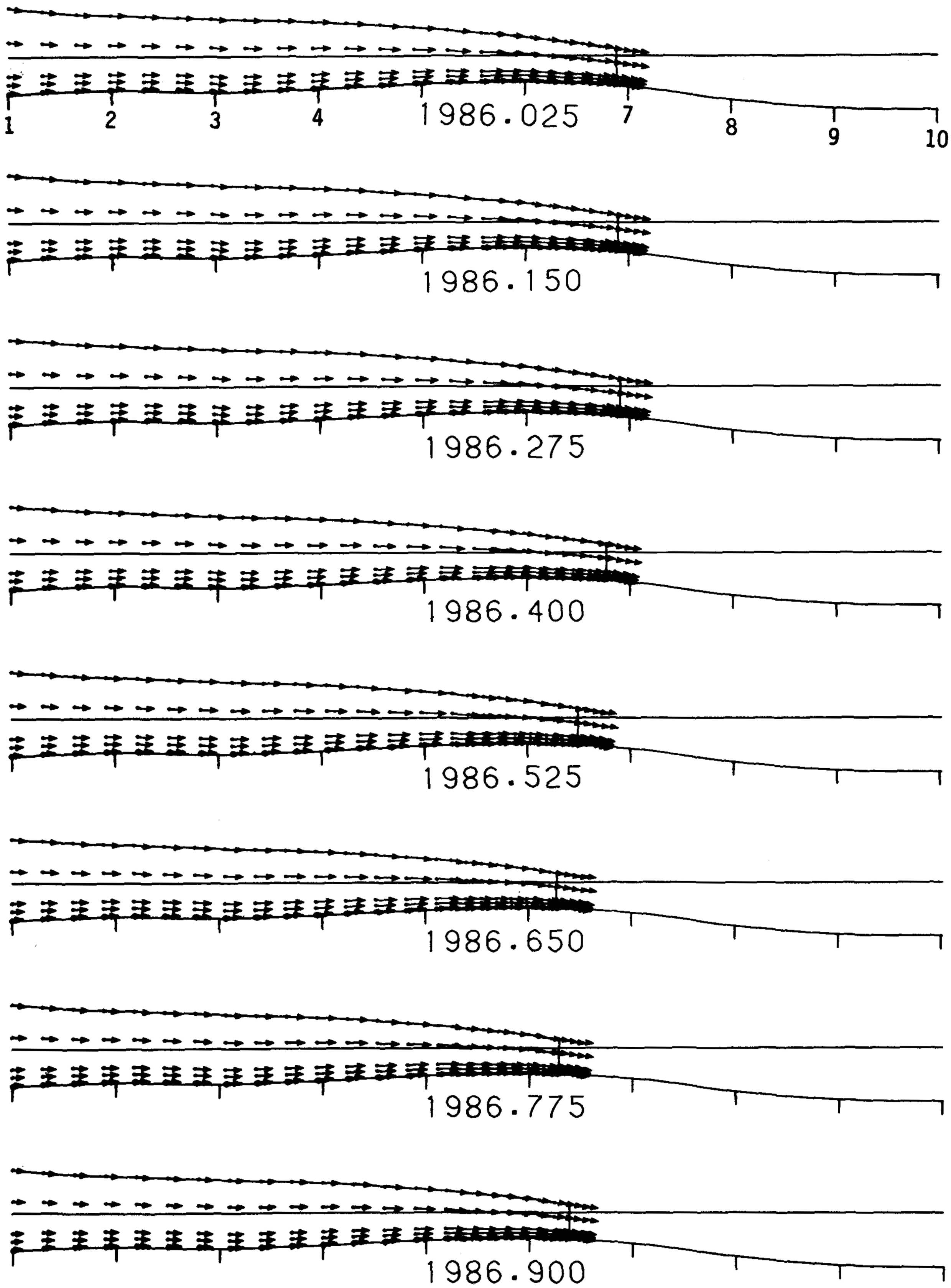


FIGURE 5 (Continued).–Future Columbia glacier behavior, projected by using modeling constant a=1165625. The individual illustrations show the flow-line profile from column locations 1 through 10 as a function of time.

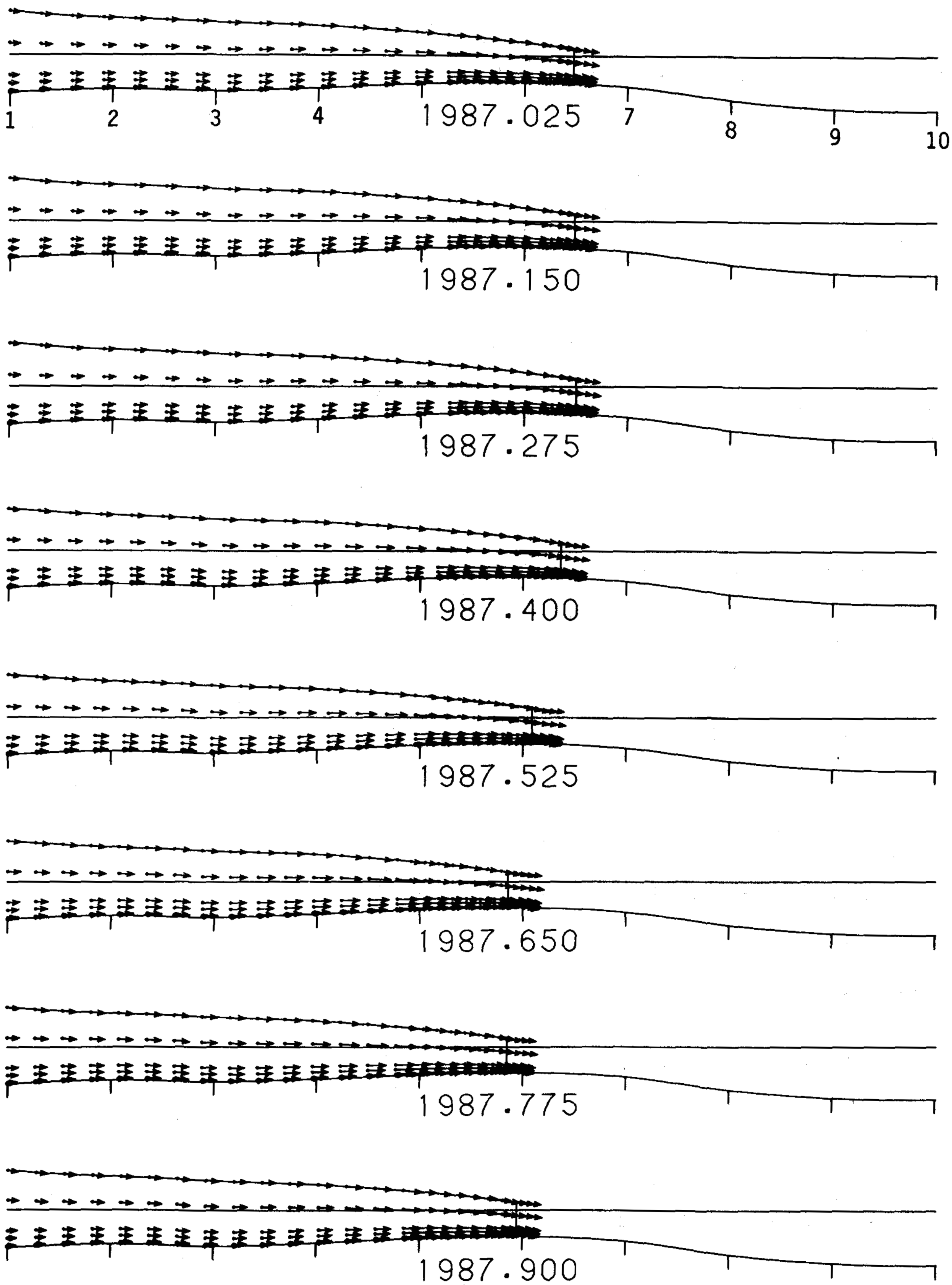


FIGURE 5 (Continued). – Future Columbia glacier behavior, projected by using modeling constant $a = 1165625$. The individual illustrations show the flow-line profile from column locations 1 through 10 as a function of time.

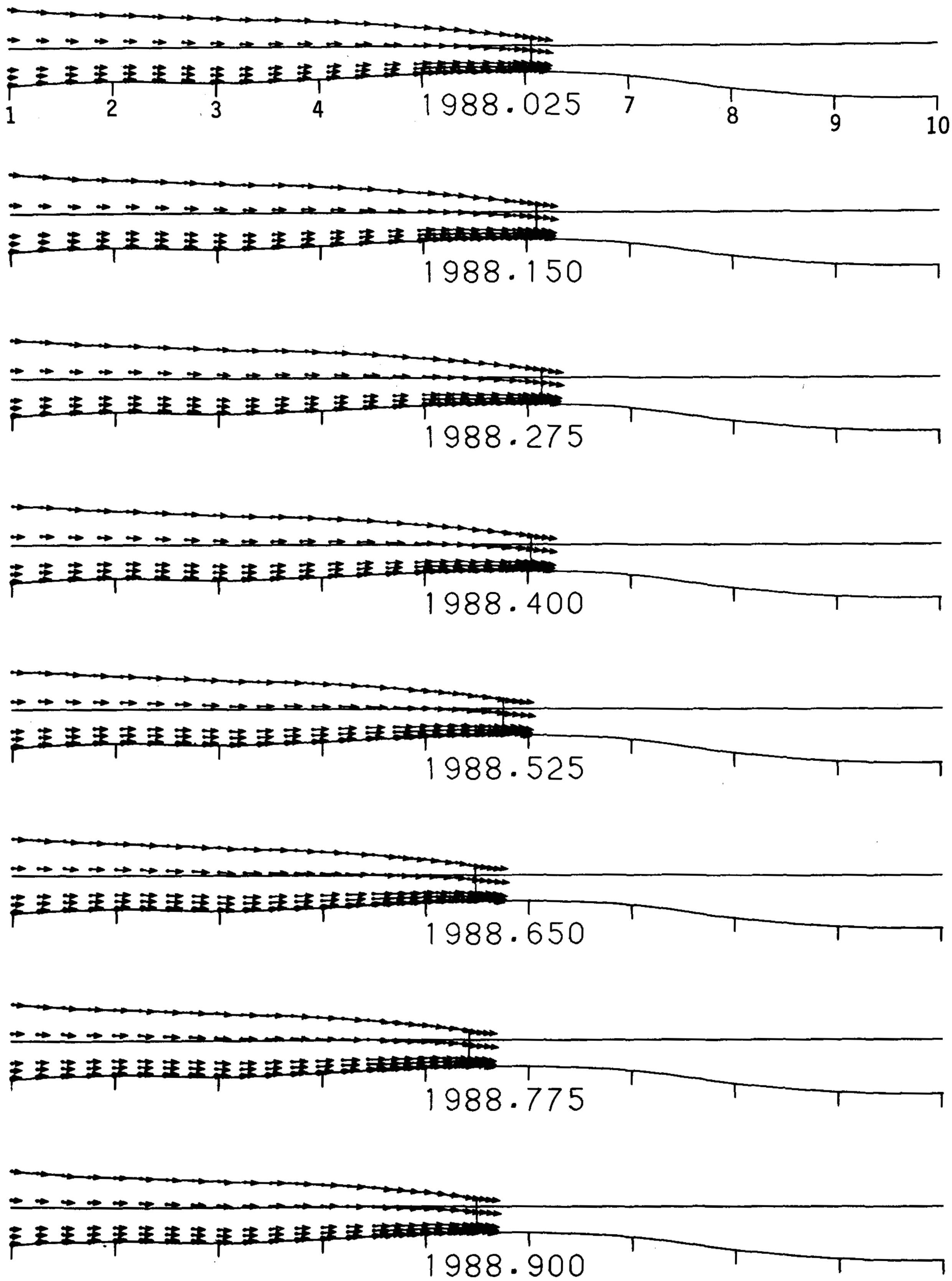


FIGURE 5 (Continued).–Future Columbia glacier behavior, projected by using modeling constant $a=1165625$. The individual illustrations show the flow-line profile from column locations 1 through 10 as a function of time.

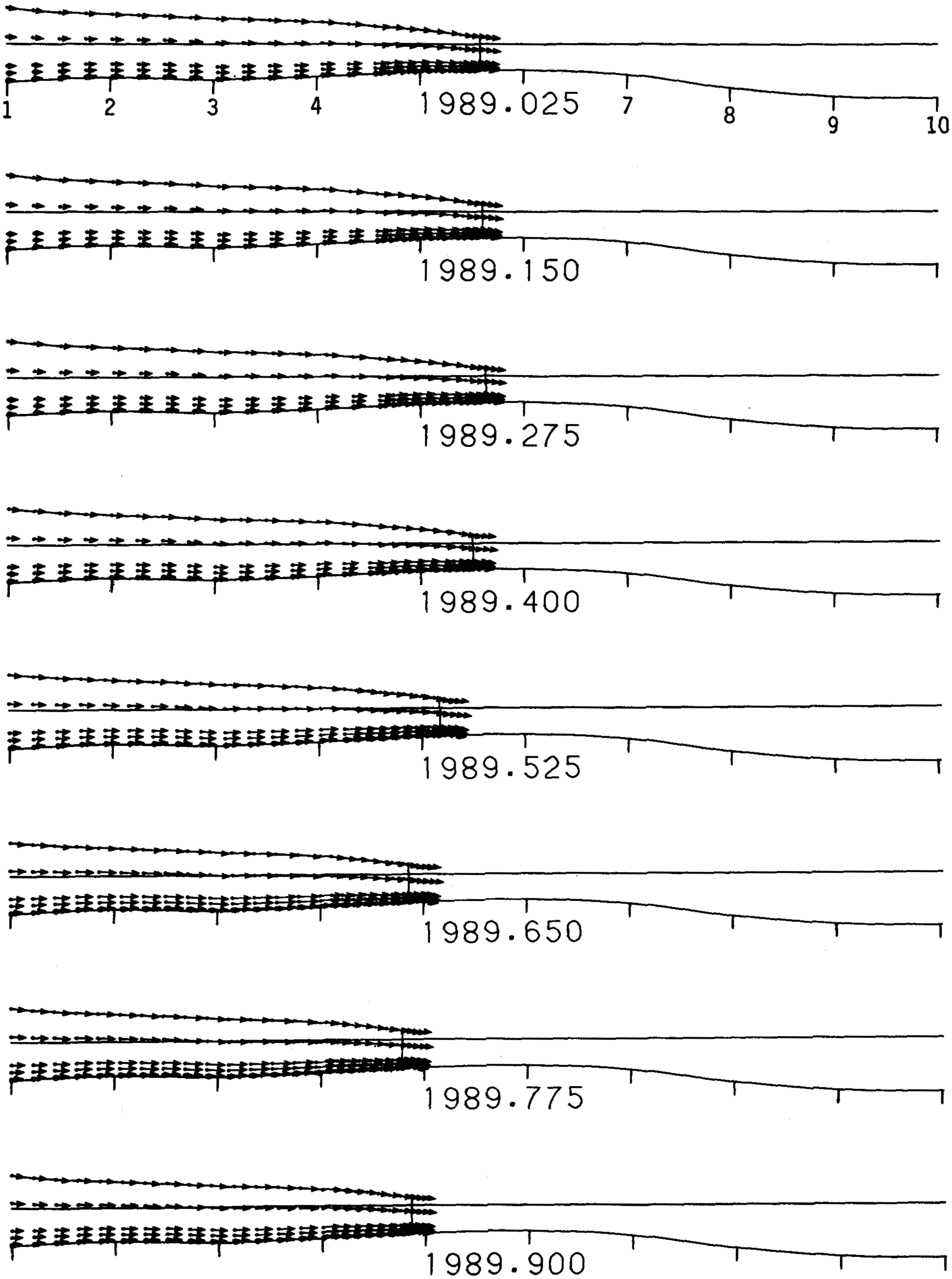


FIGURE 5 (Continued).–Future Columbia glacier behavior, projected by using modeling constant $a = 1165625$. The individual illustrations show the flow-line profile from column locations 1 through 10 as a function of time.

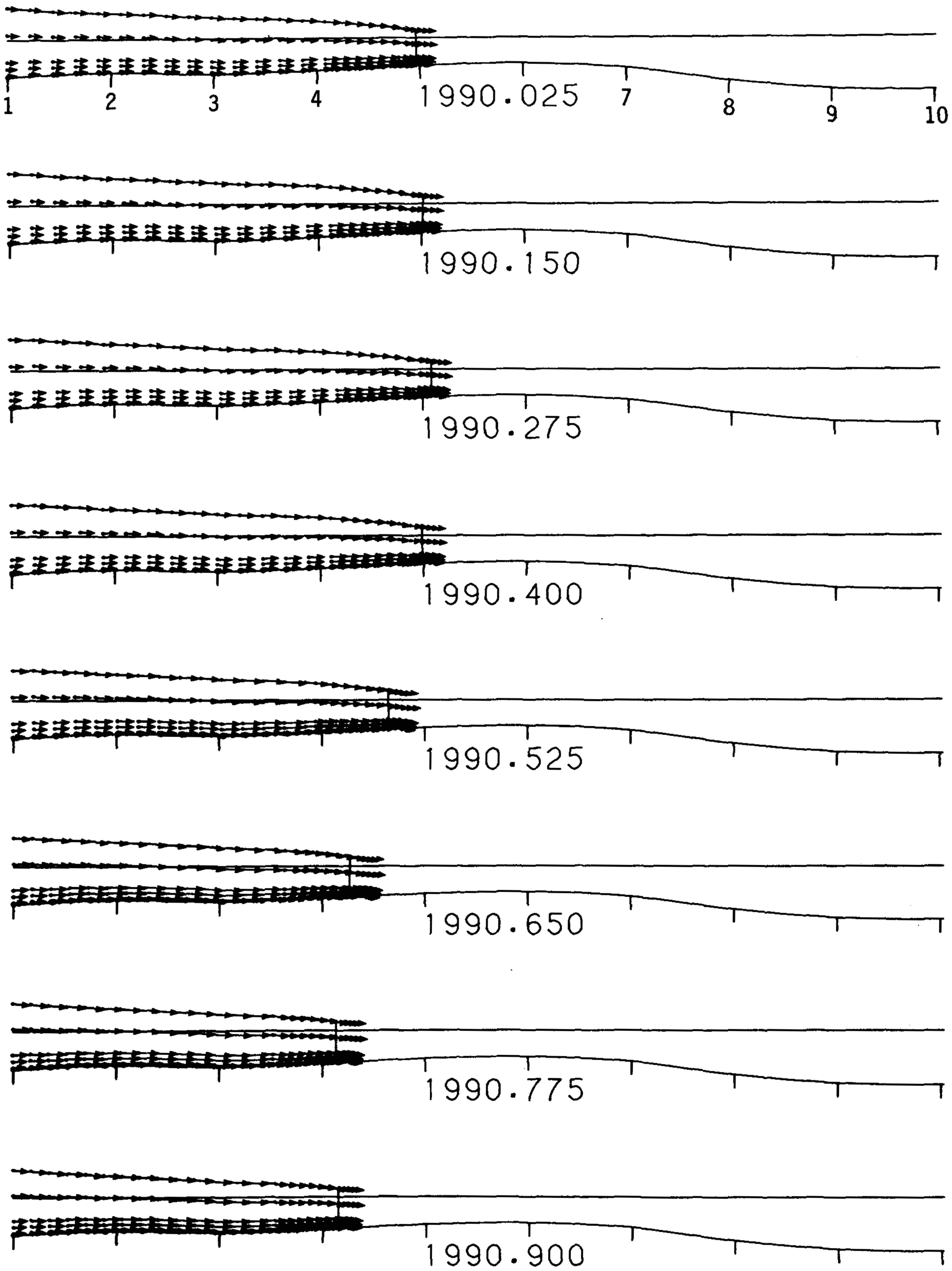


FIGURE 5 (Continued).–Future Columbia glacier behavior, projected by using modeling constant $a=1165625$. The individual illustrations show the flow-line profile from column locations 1 through 10 as a function of time.

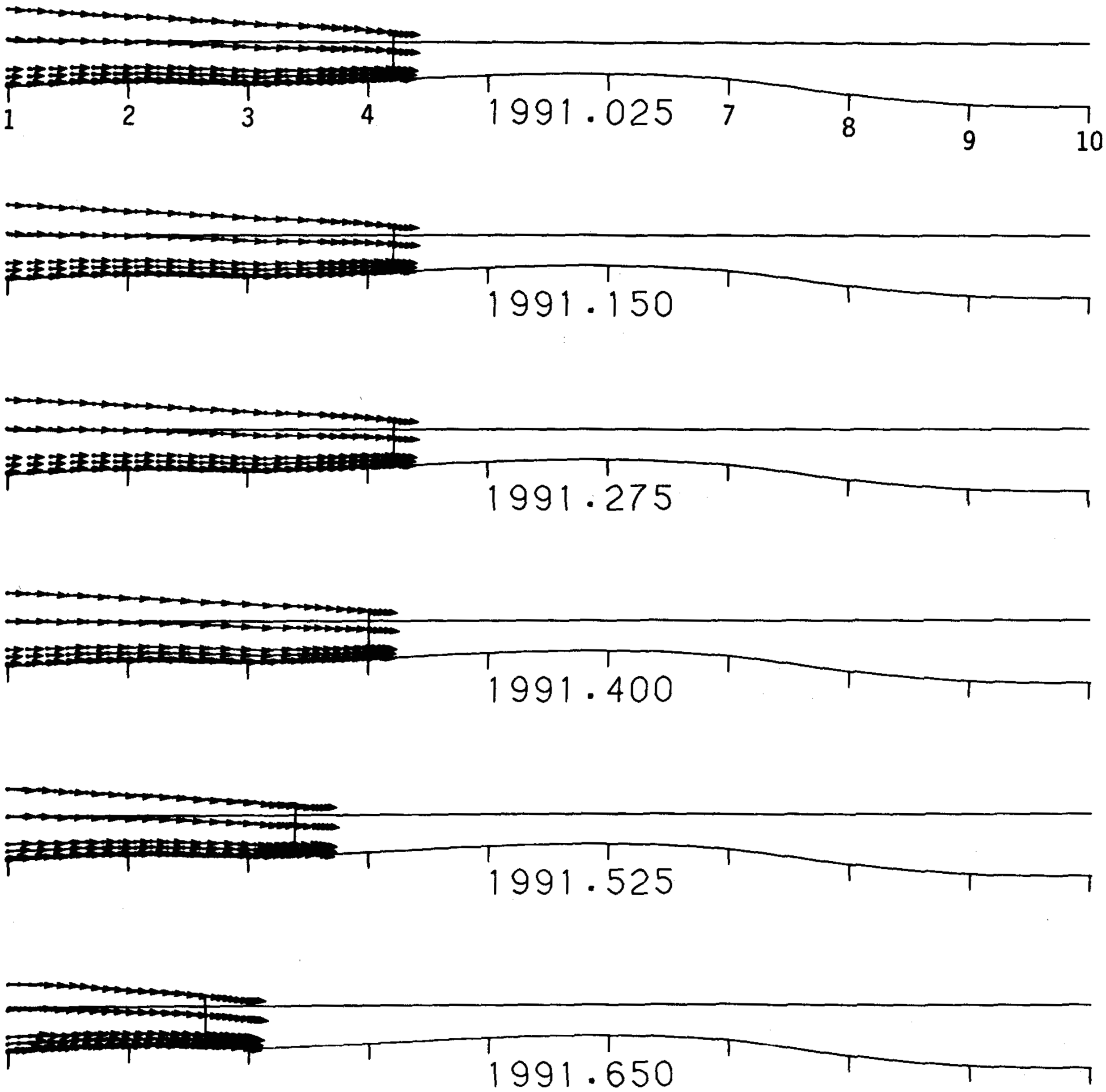


FIGURE 5 (Continued).–Future Columbia glacier behavior, projected by using modeling constant a=1165625. The individual illustrations show the flow-line profile from column locations 1 through 10 as a function of time.

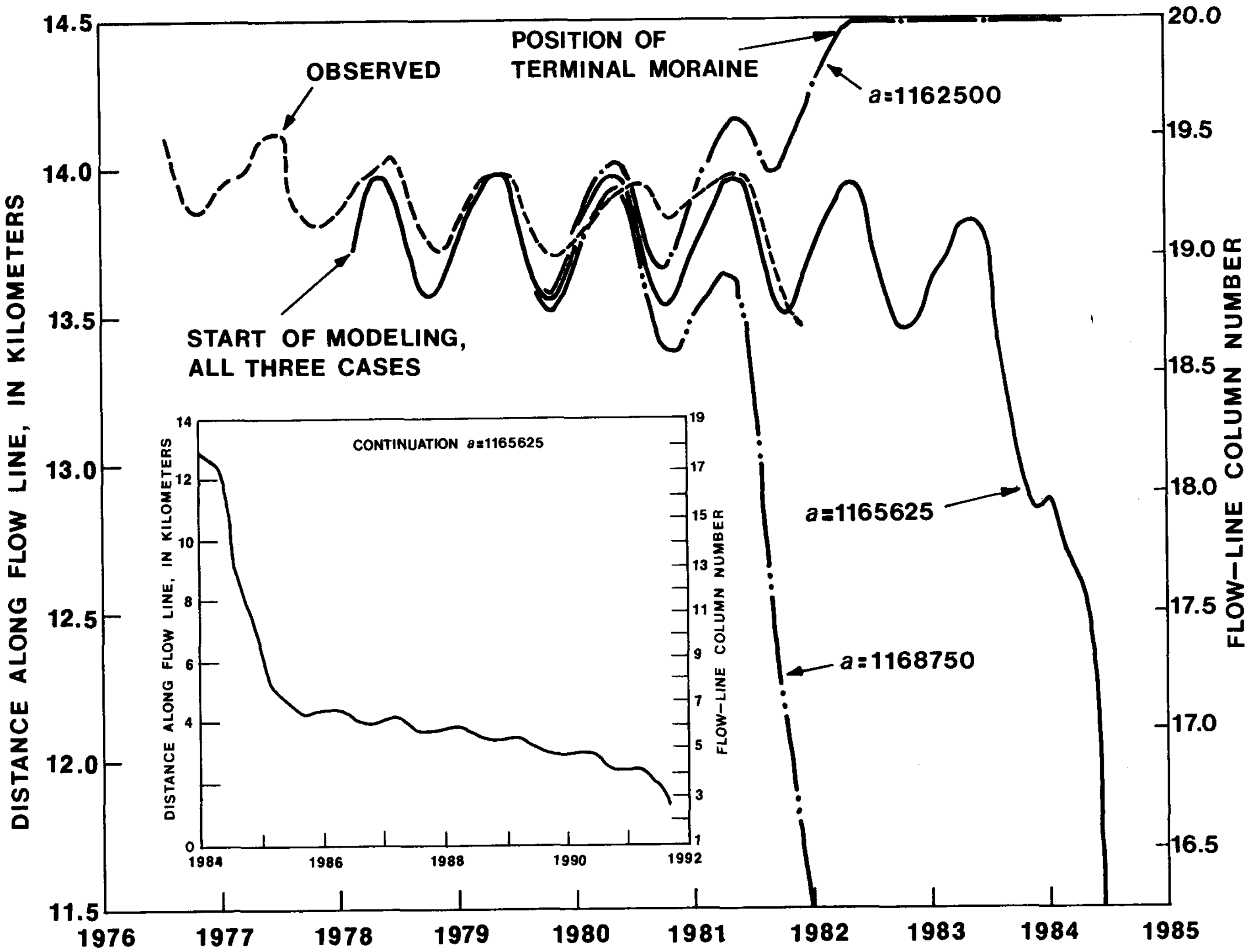


FIGURE 6. – Terminus position in time, for three values of the modeling constant a of the calving law. The observed position represents an average across the width of the terminus.

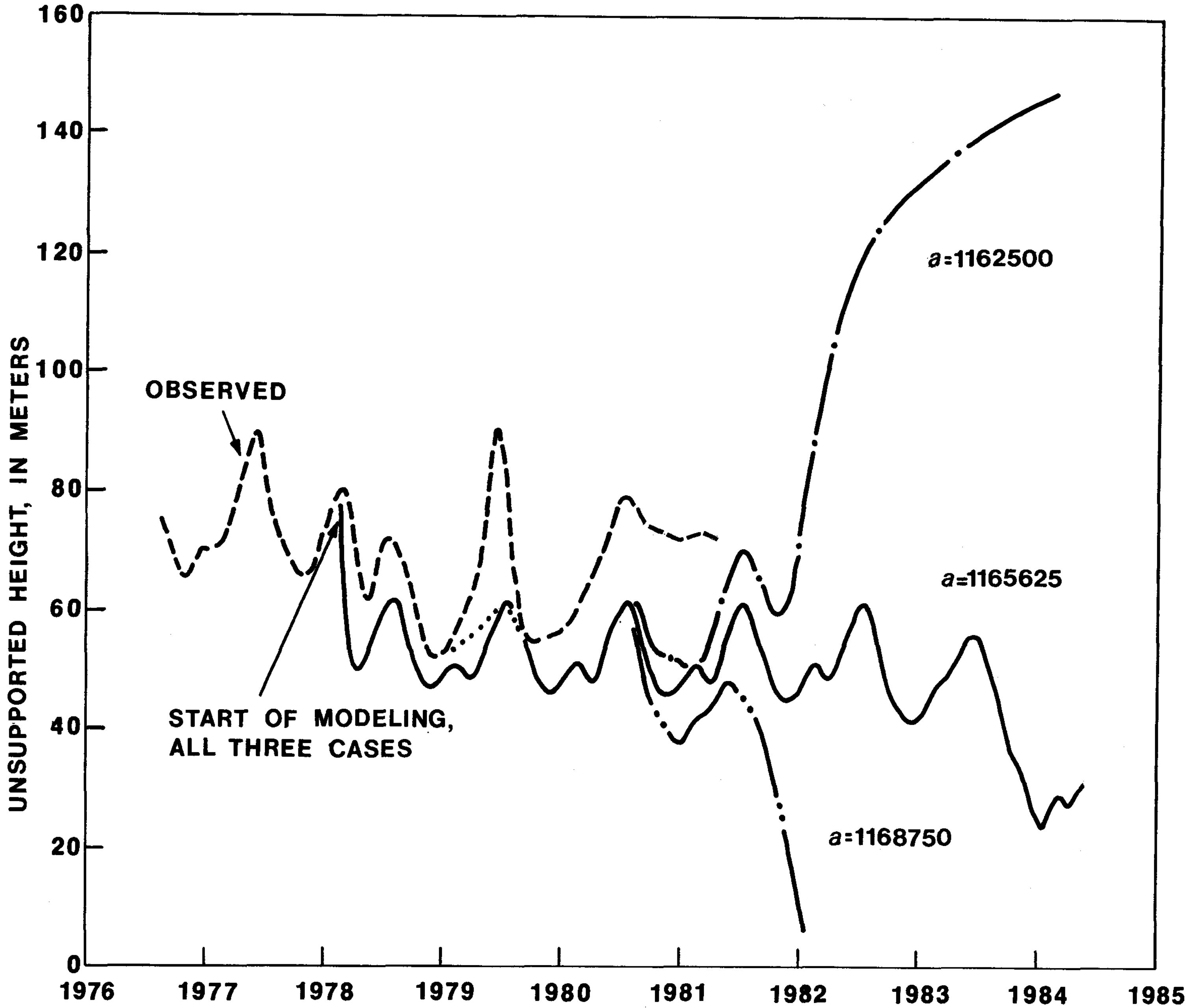
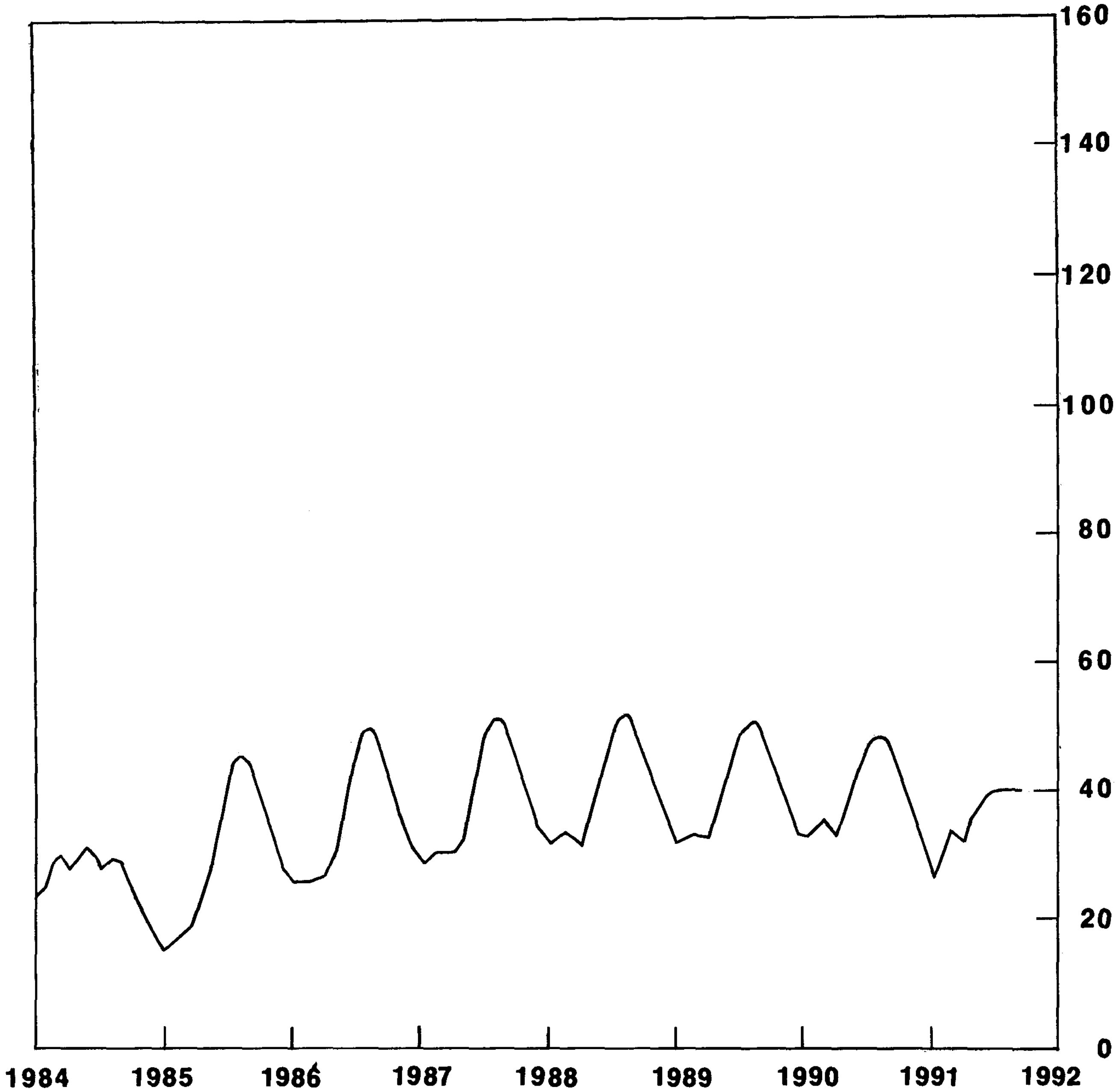


FIGURE 7.—Unsupported height, h_u, at the terminus, in time, for three values of the modeling constant a of the calving law. The observed is at the head of the embayment. The anomolous peak in this curve in early 1979 is due to a sudden, temporary switch of the location of the subglacial river discharge to a relatively undisturbed location on the terminus



with a higher ice cliff. The dotted curve represents approximately what would have resulted had this temporary shift in embayment location not occurred. The greater variability of the observed curve compared to that of the calculated curve probably is due also to the shifting of the subglacial discharge across the terminus.

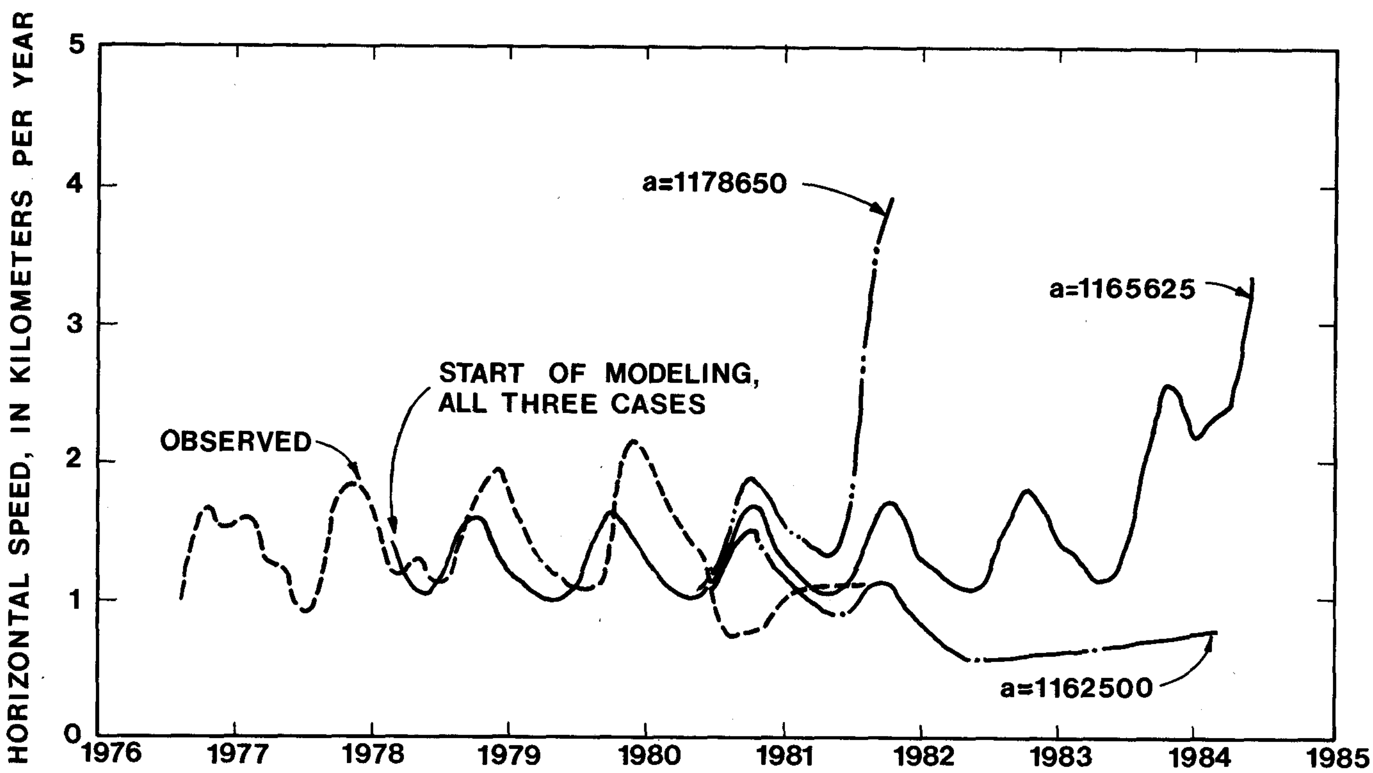


FIGURE 8.–Surface horizontal speed at flow-line-column number 16.9, in time. A phase shift of unknown origin (a three-dimensional effect?) between computed speed and observed speed indicates computed speed leading the observed speed, $(V_s)_o$, by about 1/8 year. Note the low speeds in 1980, when the retreat was interrupted.

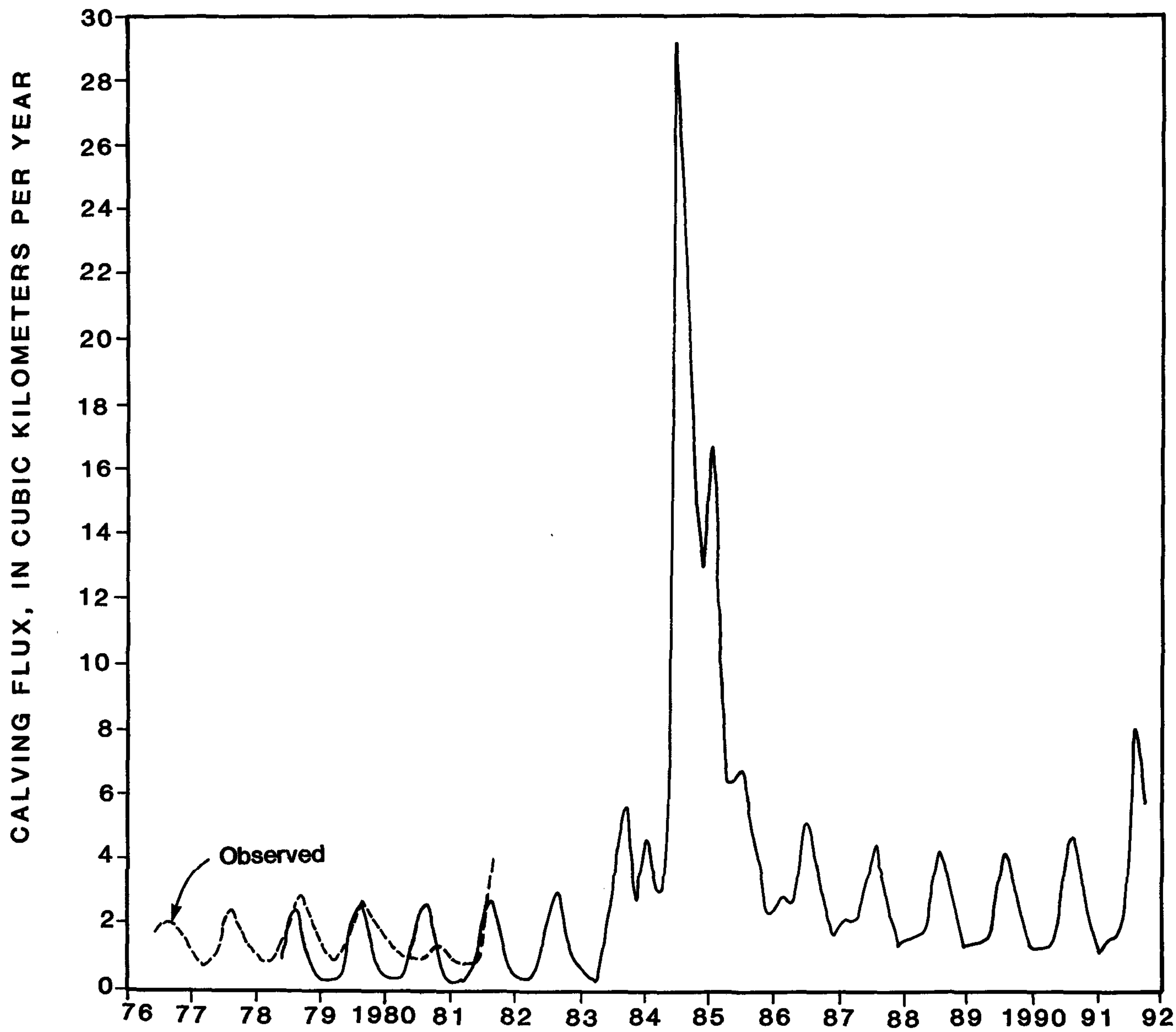


FIGURE 9.—Rate of iceberg production, in volume, projected by using modeling constant $a = 1165625$. This graph was derived by converting the two-dimensional flux of the model to three dimensions, using bed configuration and spatial velocity distributions for the real glacier.

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APPENDIX A: MATHEMATICS

The formulation used is that of Zienkiewicz (1977, p. 612, sec. 22.3.1). The $\bar{\mathbf{K}}$ and **M** matrices can be omitted as they represent contributions from kinetic effects that can be ignored in the slow movement of glaciers. Quadratic Lagrange shape functions were used for velocities, and so each element is three nodes by three nodes. Pressure shape functions were chosen to be linear in the vertical direction and quadratic in the horizontal direction, and so pressure unknowns are associated with only the top and bottom nodes of each element; that is, with nodes number 4, 7, 3, 1, 5, and 2 in figure 24.25 of Zienkiewicz (1977, p. 748). Three-by-three Gaussian intregration is used. The starting point for the element subroutine is Taylor's fluid element (Zienkiewicz, 1977, p. 755). The viscosity, μ, for ice is nonconstant, and so the resultant system of equations is solved iteratively and has a contribution to the tangent stiffness matrix (Zienkiewicz, p. 699, equation 24.2) arising from appropriate partial derivatives of μ.

TANGENT STIFFNESS MATRIX

Assume for simplicity that $\mathbf{a}^u$, $\mathbf{a}^p$ are partitioned within an element as in section 22.3.1 of Zienkiewicz (1977, p. 612). (Actually, in the program, the velocities and pressures alternate, grouped according to node number.) In Zienkiewicz' notation, the equation for ice is (see equation 22.17)

$$\mathbf{F}(\mathbf{a}) \equiv \mathbf{K}\mathbf{a}^u + \mathbf{K}^p\mathbf{a}^p + \mathbf{f}^u = \mathbf{0}, \tag{A1}$$

where

$$\mathbf{a}^T \doteq ((\mathbf{a}^u)^T, (\mathbf{a}^p)^T). \tag{A2}$$

This equation is nonlinear because μ (in **K**) is a function of the effective shear stress, and so of $\mathbf{a}^u$. To solve iteratively by Newton's method, expand to the linear term in Taylor's series:

$$\mathbf{F}(\mathbf{a}) + \mathbf{F}'(\mathbf{a})\delta\mathbf{a} \cong \mathbf{0} \tag{A3}$$

or

$$\left[\frac{\partial}{\partial \mathbf{a}^u}(\mathbf{K}\mathbf{a}^u)\right]\delta\mathbf{a}^u + \mathbf{K}^p\delta\mathbf{a}^p \cong -\mathbf{F}(\mathbf{a}). \tag{A4}$$

Here **a** is the trial, or "current," approximation of the solution of equation A1, and $\delta\mathbf{a}$ represents a correction to be applied to improve this approximation. Considering the first term in which

$$\mathbf{K}\mathbf{a}^u = [\int_\Omega \mathbf{B}^T\mathbf{D}'\mathbf{B}d\Omega]\mathbf{a}^u, \tag{A5}$$

define

$$\mathbf{G} = \frac{1}{\mu}\mathbf{B}^T\mathbf{D}'\mathbf{B}. \tag{A6}$$

Differentiating,

$$\left[\frac{\partial}{\partial \mathbf{a}^u}(\mu\mathbf{G}\mathbf{a}^u)\right]\delta\mathbf{a}^u = \left[\frac{\partial\mu}{\partial \mathbf{a}^u}\delta\mathbf{a}^u\right]\mathbf{G}\mathbf{a}^u + \mu\mathbf{G}\delta\mathbf{a}^u. \tag{A7}$$

Note that $\frac{\partial\mu}{\partial \mathbf{a}^u}$ is a row matrix, while $\partial\mathbf{a}^u$ and $\mathbf{G}\mathbf{a}^u$ are column matrices. Because of the special nature of these matrices, the order of operations in the first term on the right can be rearranged to give

$$\left[\frac{\partial}{\partial \mathbf{a}^u}(\mu\mathbf{G}\mathbf{a}^u)\right]\delta\mathbf{a}^u = \left[\mathbf{G}\mathbf{a}^u\frac{\partial\mu}{\partial \mathbf{a}^u} + \mu\mathbf{G}\right]\delta\mathbf{a}^u. \tag{A8}$$

Thus equation A4 becomes

$$\left[\int_\Omega \mathbf{G}\mathbf{a}^u\frac{\partial\mu}{\partial \mathbf{a}^u}d\Omega + \mathbf{K}\mathbf{a}^u\right]\delta\mathbf{a}^u + \mathbf{K}^p\delta\mathbf{a}^p \cong -\mathbf{F}(\mathbf{a}). \tag{A9}$$

The integral represents the contribution due to the nonconstant viscosity, μ. To express it in workable form, note that

$$\frac{\partial\mu}{\partial \mathbf{a}^u} = \frac{\partial\mu}{\partial d^2}\,\frac{\partial d^2}{\partial \mathbf{a}^u}, \tag{A10}$$

where

$$d = (\tfrac{1}{2}d_{ij}d_{ij})^{1/2} \tag{A11}$$

(summation convention on subscripts) is the effective shear deformation rate. Using the notation of Zienkiewicz, section 22.3.1, and writing column matrix **d** in place of d_{ij},

$$\mathbf{d} = \mathbf{B}\mathbf{a}^u, \tag{A12}$$

we have

$$d^2 = \tfrac{1}{4}(\mathbf{B}\mathbf{a}^u)^T\mathbf{E}(\mathbf{B}\mathbf{a}^u), \tag{A13}$$

where

$$\mathbf{E} \equiv \frac{1}{\mu}\mathbf{D}. \tag{A14}$$

Taking the derivative,

$$\frac{\partial d^2}{\partial \mathbf{a}^u}\delta\mathbf{a}^u = \tfrac{1}{4}[(\delta\mathbf{a}^u)^T\mathbf{B}^T\mathbf{E}\mathbf{B}\mathbf{a}^u + \mathbf{a}^{uT}\mathbf{B}^T\mathbf{E}\mathbf{B}\delta\mathbf{a}^u], \quad \text{(A15)}$$

or rearranging,

$$\frac{\partial d^2}{\partial \mathbf{a}^u}\delta\mathbf{a}^u = [\tfrac{1}{2}\mathbf{a}^{uT}\mathbf{B}^T\mathbf{E}\mathbf{B}]\delta\mathbf{a}^u, \quad \text{(A16)}$$

that is,

$$\frac{\partial d^2}{\partial \mathbf{a}^u} = \tfrac{1}{2}\mathbf{a}^{uT}\mathbf{B}^T\mathbf{E}\mathbf{B} = \tfrac{1}{2}(\mathbf{G}\mathbf{a}^u)^T. \quad \text{(A17)}$$

From equations 1, 6, and A11, the ice viscosity, μ, can be written in terms of the effective deformation rate, d, as

$$\mu = \frac{(d^2)^{((1-n)/2n)}}{2A^{1/n}}. \quad \text{(A18)}$$

This yields

$$\frac{\partial \mu}{\partial d^2} = \frac{(1-n)d^{((1/n)-3)}}{4nA^{1/n}} \quad \text{(A19)}$$

Thus, the integral term in equation A9 – the contribution in Newton's method due to nonconstant μ – is

$$\left[\int_\Omega \tfrac{1}{2}\frac{\partial \mu}{\partial d^2}(\mathbf{G}\mathbf{a}^u)(\mathbf{G}\mathbf{a}^u)^T d\Omega\right]\delta\mathbf{a}^u. \quad \text{(A20)}$$

The column matrix

$$\mathbf{Q} \equiv \mathbf{G}\mathbf{a}^u = \mathbf{B}^T\sigma' \quad \text{(A21)}$$

(defining $\mathbf{Q}$ as found in the element subroutine, not the $\mathbf{Q}$ of equation 24.10 of Zienkiewicz (1977, p. 710)) is evaluated by using the "current" velocities, $\mathbf{a}^u$, of the iterative Newton's method process. $\mathbf{Q}$ must be calculated as part of accounting for "current" internal forces (Zienkiewicz, equation 24.2, p. 699), so the dependence of the tangential matrix on μ can be obtained with little additional effort.

Some distribution of μ must be chosen initially to start the iterative process. In the program, this initial μ is assumed to be given by equation 6 with effective shear stress, τ, identically 1 bar throughout the ice mass.

UPPER FREE-SURFACE DYNAMICS

To follow the motion of the glacier surface in time, the ice velocity along the surface, Γ_s, is resolved into generally nonorthogonal components along the surface and in the vertical direction; this vertical component is called the emergence velocity, V_e. If γ is the angle between horizontal and an upward normal to the surface, define

$$\mathbf{s}^T = (\text{ctn}\,\gamma, 1), \quad \text{(A22)}$$

then

$$V_e = \mathbf{s}^T\mathbf{u} = \mathbf{s}^T\mathbf{N}^u\mathbf{a}^u. \quad \text{(A23)}$$

For V_e evaluated on the surface, the only nonzero elements in $\mathbf{N}^u$ are those which correspond to surface nodes. Let $\dot{h}$ represent the (unknown) rate of thickness change and suppose that

$$\dot{h} = \mathbf{N}^h\mathbf{a}^h, \quad \text{(A24)}$$

where the scalar shape function elements in $\mathbf{N}^h$ correspond to scalar velocity shape function elements for the surface nodes, for evaluation on the surface only; that is, they correspond to the nonzero elements of $\mathbf{N}^u$ in equation A23. If $\dot{b}$ is the balance rate, then at any point on the surface,

$$\dot{h} - V_e - \dot{b} = 0. \quad \text{(A25)}$$

To obtain a suitable numerical equation, use the Galerkin process and premultiply equation A25 by $(\delta\dot{h})^T$, then integrate over the surface:

$$(\delta\mathbf{a}^h)^T\int_{\Gamma_s}\mathbf{N}^{hT}(\dot{h} - V_e - \dot{b})\,d\Gamma_s = 0. \quad \text{(A26)}$$

Define

$$\mathbf{K}_{ij}^h = \alpha\int_{\Gamma_s}(\mathbf{N}_i^h)^T\mathbf{N}_j^h d\Gamma_s,$$

$$\mathbf{K}_{ij}^u = -\alpha\int_{\Gamma_s}(\mathbf{N}_i^h)^T\mathbf{s}^T\mathbf{N}_j^u\,d\Gamma_s, \quad \text{(A27)}$$

$$\mathbf{f}_i^h = -\alpha\int_{\Gamma_s}(\mathbf{N}_i^h)^T\dot{b}\,d\Gamma_s,$$

where $\dot{b}$ is actually stored for the surface nodes and interpolated along the surface as $\dot{h}$ is in equation A24 and where α is a constant to be explained in the next paragraph. Thus, equation A26 is

$$\mathbf{K}^h\mathbf{a}^h + \mathbf{K}^u\mathbf{a}^u + \mathbf{f}^h = 0. \quad \text{(A28)}$$

The equation which replaces equation 22.22 of Zienkiewicz (1977, p. 613) is as follows:

$$\begin{bmatrix}\mathbf{K}^h & \mathbf{K}^u & 0\\ \mathbf{K}^{uT} & \mathbf{K} & \mathbf{K}^p\\ 0 & \mathbf{K}^{pT} & 0\end{bmatrix}\begin{Bmatrix}\mathbf{a}^h\\ \mathbf{a}^u\\ \mathbf{a}^p\end{Bmatrix} + \begin{Bmatrix}\mathbf{f}^h\\ \mathbf{f}\\ 0\end{Bmatrix} = 0. \quad \text{(A29)}$$

$\mathbf{K}^{uT}$ was added to the stiffness matrix to preserve symmetry, and the value assigned to constant α of equations A27 was chosen to be a very small number (10^{-16}) so that the $\mathbf{K}^{uT}$ would not significantly alter the second (set of) rows of equation A29.

Here the unknowns $\mathbf{a}^h$, $\mathbf{a}^u$, and $\mathbf{a}^p$ have been partitioned into groups for simplicity, but, in the program, they alternate, grouped by node number. The unknowns $\mathbf{a}^h$ occupy some of the third nodal degree of freedom (DOF) not needed for pressure unknowns $\mathbf{a}^p$ (since pressure is only linear in the vertical direction). The third DOF of the second node from the surface in a column, thus, is assigned the unknown $\mathbf{a}_i^h$, which then represents the rate of surface altitude change at the current column location.

The tangent stiffness matrix has $\mathbf{K}^h$, $\mathbf{K}^u$, and $\mathbf{K}^{uT}$ added in the same locations as in equation A29. The right side of the Newton's method equation must include the contribution arising from "current" V_s, $\dot{b}$, and $\dot{h}$,

$$\mathbf{P}_i^h = \alpha \int_{\Gamma_s} (\mathbf{N}_i^h)^T (V_s + \dot{b} - \dot{h}) d\Gamma_s. \qquad \text{(A30)}$$

(Caution: **P** as defined by Taylor in the element routines is the entire right side of the Newton's method equation; that is, it is $-\mathbf{f} - \mathbf{P}$ of equation 24.2 or equation 18.10 (Zienkiewicz, 1977, p. 699, 453).)

SURFACE TRACTIONS AND BODY FORCES

Water and atmospheric pressure (surface tractions) and gravity (a body force) are added to the equations, as in equation 22.16 of Zienkiewicz (1977, p. 612).

UPDATING THE MESH

If h_s denotes the ice surface altitude and h_b is the bed altitude and

$$h = h_s - h_b \text{ meters}, \qquad \text{(A31)}$$

the eight nodes within a vertical column have been chosen to be at locations h_s, $h_s - 0.4\ (h-10)$, $h_s - 0.8\ (h-10)$, $h_s - 0.9\ (h-10)$, $h_b + 10$, $h_b + 10$, $h_b + 5$ and h_b, with a greater concentration of nodes near the bed where deformation rates are higher. (In an infinitely wide slab glacier with $n = 3$, deformational velocity varies through the thickness as a quartic; the program would approximate this quartic with two quadratics.)

The upper free surface and calving face are updated as in section 22.6 of Zienkiewicz (1977, p. 623), using $\mathbf{a}^h$ calculated as described in the section of Appendix A entitled "Upper Free-Surface Dynamics" and the calving speed from the calving law of equation 8. Because of large deformations, the entire mesh is updated at each step. The updating is done by calculating glacier length at each time step and then by requiring that each column of nodes be at the same fractional location within this new length as it was initially. Nodes are then distributed within each column as a fraction of the new thickness, as described in the preceding paragraph.

FUTURE MODIFICATION

The program was written to allow easy future modification to three space dimensions; a large part of this change can be done by simply replacing two-dimensional shape functions by three-dimensional ones.

APPENDIX B: USER INSTRUCTIONS FOR THE COMPUTER PROGRAM

The general purpose finite element program resembles closely that described by Taylor in chapter 24 of Zienkiewicz (1977); only relevant changes and specialized modules are described in this appendix. The program now (1980) available from Taylor contains a data prerocessor that converts free format input to fields always of length 10 – if an integer is expected, the field is I10; if a FORTRAN real is expected, the field is F10.0.

COORDINATE DATA

The program assumes that nodes are specified in vertical columns. Each column is, to some extent, considered a unit. The sample run of Appendix C (p. B34–B46) uses columns of 8 nodes (the user can select more elements per column if he chooses). The third and fourth nodes from the bottom in a column are at the same location on the column and have the velocity DOF tied together, allowing only a pressure discontinuity to accommodate the discontinuity in the A flow law parameter at that point. Extra "dummy" columns can be specified beyond the initial terminus position to accommodate an advancing glacier. The NUMNP on the control card should be the total number of nodes including these extras. The true terminus position is as specified on the material property data card.

MATERIAL PROPERTY DATA

Columns	Description	Variable
01 to 10	A of ice-flow law	D(1,1)
11 to 20	n of ice-flow law	D(2,1)
21 to 30	Uppermost element in terminal face	D(3,1)
31 to 40	Number of nodes in each column	D(4,1)

BOUNDARY RESTRAINT DATA

Boundary Condition Cards – FORMAT(8I10). – Use only 0, +1, and −1 for the constraint conditions that Taylor describes. A boundary code describing tieing is the node number of a preceding like-numbered DOF. For example, to tie DOF 1 and DOF 2 of node 14 to DOF 1 and DOF 2 of node 13, specify the following boundary condition codes:

14	8	13	13
198	0	197	197

The tieing uses the generation increment also to tie the first two degrees of freedom for node 22 to 21, node 30 to 29, . . . , node 198 to 197. When tieing is specified, only the DOF of the node tied to is retained in the system of equations. Do not tie degrees of freedom that have specified displacements.

Unused degrees of freedom should have displacements specified. Thus, the middle row of nodes in each element, except for the surface elements, should have specified displacements for the third DOF, because pressure is only modeled linearly in the vertical direction. (The middle row of nodes of the surface elements have the $\dot{h}$ unknown as the third DOF.) Also, the extra "dummy" nodes beyond the initial terminus position should have all degrees of freedom with displacements specified.

NODAL FORCED BOUNDARY DATA

Boundary forces F(3,·) corresponding to unused degrees of freedom in a column are used to store the balance rate, $\dot{b}$, and sliding flow law parameter, A_b. These are considered to be at the location of the column when dynamic modeling changes the column position. Within a column, storage is as follows:

"Current" parameters F(3,·)	*Location in column*
–	1 (surface)
–	2
–	3
$\dot{b}$	4
–	5
. . .	. . .
–	NDEEP-2
A_b	NDEEP-1
–	NDEEP (bed)

For example, if the second node from the bottom in a column is node number 55, the sliding flow law parameter, A_b, for the current position of the column is stored in F(3,55).

NODAL TEMPERATURE DATA

In a manner similar to column storage of parameters in F(3,·), nodal "temperatures," T(·), are used to store information corresponding to the initial configuration of the ice mass. Note the distinction that the T(·) refer to fixed initial column coordinates, while the F(3,·) refer to current dynamic column coordinates. Within a column, storage is as follows:

"Initial" parameters, T(*)	*Location in column*
X_b	1 (surface)
Y_b	2
Z_b (future use)	3
$\dot{b}$	4
$\dot{h}$ (future use)	5
...	...
V_s	NDEEP-2
A_b	NDEEP-1
—	NDEEP (bed)

Here X_b, Y_b, and Z_b are the bed coordinates; $\dot{b}$, the balance rate; $\dot{h}$, the rate of surface altitude change; V_s, the horizontal surface speed; and A_b, the sliding flow law parameter–all representing measured or estimated values for the initial column position. For example, if the second node from the surface in a column is node number 50, the y coordinate of the bed at the initial column position is stored in T(50).

TANF MACROINSTRUCTION

A few additional macroinstructions have been added to Taylor's set. TANF simply combines the functions of his TANG and FORM. Proceeding this way was more efficient for the case at hand and is, in fact, now required by the element subroutine ELMT04 for ice flow rather than the TANG and FORM routines.

REST AND SAVE MACROINSTRUCTIONS

Program restart capability is provided through REST and SAVE macroinstructions. SAVE writes the node number, nodal coordinates, and nodal degrees of freedom to FORTRAN file 15 with

```
FORMAT(I16,4(1PE16.9))
```

or

```
FORMAT(I16,4(1PE16.9)/5(1PE16.9))
```

depending on the number of coordinates and degrees of freedom. Macro REST reads this file as FORTRAN file 16. If

```
REST          1.
```

is specified, where the 1. is in columns 21 to 30, the coordinates, $\dot{b}$, A_b, and the degrees of freedom are updated from the restart file. If 0. is given in place of the 1., the coordinates, $\dot{b}$ and A_b are left as on the initial input data, and only the degrees of freedom are updated.

UPNO MACROINSTRUCTION

The mesh is updated as in section 22.6 of Zienkiewicz (1977, p. 623) by using the macroinstruction (UPNO (mnemonically, "update nodes"). The upper surface is adjusted according to ice velocity vectors and balance rate, and the calving face is adjusted according to the calving law of equation 7.

PLOT MACROINSTRUCTION

The PLOT macroinstruction produces a plot giving surface, bed, and calving face outlines and velocity vectors. The plot routine expects scaling information immediately following a SCAL macro card that is placed after the END macro that terminates MACR processing. The scaling parameters are as follows:

Columns	*Description*	*Variable*
01 to 10	Minimum x-coordinate in plot	XMIN
11 to 20	Maximum x-coordinate in plot	XMAX
21 to 30	Minimum y-coordinate in plot	YMIN
31 to 40	Maximum y-coordinate in plot	YMAX
41 to 50	Scale factor in units per inch	SCALE
51 to 60	Vector scale factor in speed units per inch	VSCALE
61 to 70	Arrowhead length in inches	AHLEN
71 to 80	Plot rotation, 0 or 90 degrees (only)	ROT

This scaling parameter card is followed by an END macro; that is, the plot routine reads the SCAL card, the parameter card, and an END card.

APPENDIX C: NEW SUBROUTINES

The following subroutines were written by the author specifically to treat the modeling of glaciers.

```
      SUBROUTINE ELMT04(D,UL,XL,IX,TL,S,P,NDF,NDM,NST,ISW)
C
C.... TWO DIMENSIONAL ELEMENT FOR ICE FLOW
C
      COMMON /CDATA/ O,HEAD(20),NUMNP,NUMEL,NUMMAT,NEN,NEQ,IPR
      COMMON /ELDATA/ DM,N,MA,MCT,IEL,NEL
      COMMON /RSTART/ ISTART
      COMMON /JACOBI/ XS(2,2),SX(2,2)
      COMMON M(1)
      COMMON /POINTR/ N0,N1,N2,N3,N4,N5,N6,N7,N8,N9,N10,N11,N12,N13,
     * N14,NE
      LOGICAL SFLAG
      DIMENSION XL(NDM,1),UL(NDF,1),P(NDF,1),S(NST,1),D(1),V(3),DV(2,2)
     1,SHP(3,9),IX(1),SG(9),TG(9),WG(9),SHPP(3,9),TL(1)
      DIMENSION SG1D(3),TG1D(3),WG1D(3),IANODE(9),IXP(9),INTERP(9)
      DIMENSION ITOP(3),MID(3),Q(27),ANODE(3)
C.... IANODE GIVES VISCOSITY "A" INTERPOLATION (REDEFINITION OF NODES).
C.... INTERP YIELDS PRESSURE INTERPOLATION (1=NODE USED, 0=NOT).
      DATA IANODE/1,2,2,1,3,2,3,1,3/,INTERP/1,1,1,1,1,0,1,0,0/
      DATA ITOP/3,4,7/,MID/6,8,9/,Q/27*0./
      GO TO (1,2,3,4,5,3),ISW
C.... INPUT/OUTPUT FLUID PROPERTIES
C.... LINT1D IS THE NUMBER OF INTEGRATION POINTS IN EACH DIRECTION
C.... D(1) = A (ICE), D(2) = N
C.... D(3) IS THE LOWEST NUMBERED NODE IN THE CALVING FACE -
C.... ALL NODES FROM D(3) TO NUMNP ARE ASSUMED TO BE IN THE FACE.
C.... D(4) IS THE NUMBER OF NODES IN A VERTICAL COLUMN THROUGH DEPTH.
    1 READ(5,1000) (D(I),I=1,4)
      WRITE(6,2000) (D(I),I=1,4)
      LINT1D = 3
      LINT = 0
      ISTART = 0
      F1EPS = 1./D(2)
      F2EPS = 1./D(2) - 1.
      RHOG = 0.0882
      RHOGW = 0.0980
      ATMOSP = 1.013
      ALPHA = 1.E-16
      NFACE = D(3) + 0.1
      NDEEP = D(4) + 0.1
```

```
      DO 20 I=1,9
      IXP(I) = 0
      IF(INTERP(I).EQ.1)NELPC=I
   20 SHPP(3,I) = 0.
      RETURN
2     RETURN
C.... COMPUTE SYMMETRIC TANGENT STIFFNESS AND OUT OF BALANCE FORCES.
C.... USE TANF, NOT TANG AND FORM -- ONLY ONE PASS PER ELEMENT.
    3 ISTART = ISTART + 1
C.... REDEFINE NUMNP TO EXCLUDE "EXTRA" NODES BEYOND CALVING FACE.
      NUMNP = NFACE + NDEEP - 1
      L = LINT1D
      IF(L*L.NE.LINT) CALL PGAUSS(L,LINT,SG,TG,WG)
      DO 65 L=1,LINT
      CALL SHAPE(SG(L),TG(L),XL,SHP,XSJ,NDM,NEL,IX,.FALSE.)
      XSJWG = XSJ*WG(L)
C.... CALL PRESSURE SHAPE FUNCTIONS -- .TRUE. BECAUSE PRESSURE
C.... GRADIENT NOT NEEDED, AND WOULD NOT BE CORRECT ANYWAY
C.... BECAUSE NOT ISOPARAMETRIC.
      NELP = MIN0(NELPC,NEL)
      DO 25 I=1,NELPC
   25 IXP(I) = IX(I)*INTERP(I)
      CALL SHAPE(SG(L),TG(L),XL,SHPP,XSJP,NDM,NELP,IXP,.TRUE.)
C.... COMPUTE VELOCITIES, VELOCITY GRADIENTS, PRESSURES = V(3),
C.... AND FLOW LAW PARAMETER A (ENTERED AS NODAL "TEMPERATURE").
C.... USE ONLY LOWER NODES TO ALLOW VISCOSITY DISCONTINUITY IN DEPTH,
C.... BUT STILL RETAIN CONTINUITY ALONG FLOW DIRECTION.
      DO 32 I = 1,2
      V(I) = 0.
      DO 31 K = 1,NEL
31    V(I) = V(I) + SHP(3,K)*UL(I,K)
      DO 32 J = 1,2
      DV(I,J) = 0.0
      DO 32 K = 1,NEL
32    DV(I,J) = DV(I,J) + SHP(J,K)*UL(I,K)
      V(3) = 0.
      DO 280 K=1,NELP
  280 V(3) = V(3) + SHPP(3,K)*UL(3,K)
      A = 0.
C.... CHECK IF AT BED.
      IF(MOD(IX(1),NDEEP).NE.0)GO TO 283
C.... COMPUTE "A" FLOW LAW COEFFICIENT ALONG BED TO SIMULATE SLIDING.
      DO 281 K=1,3
  281 CALL VALUES(ANODE(K),M(N10),3,IX(MID(K)),NDF)
      DO 282 K=1,NEL
  282 A = A + SHP(3,K)*ANODE(IANODE(K))
  283 A = AMAX1(D(1),A)
C.... COMPUTE EFFECTIVE STRAIN RATE EPSEFF
      EPSEFF = 0.
      DO 290 I=1,2
```

```
      DO 290 J=1,2
  290 EPSEFF = EPSEFF + (DV(I,J) + DV(J,I))**2
      EPSEFF = SQRT(EPSEFF/8.)
C.... NOW COMPUTE VISCOSITY AMU -- USE AMUCON IF
C.... FIRST PASS (ALL ELEMENTS) THROUGH TANF -- INDICATED
C.... BY FLAG ISTART.LE.NUMEL.
C.... FOR FIRST PASS USE VISCOSITY AT BED WITH TAUBED = 1.
C.... DMU IS DERIVATIVE OF VISCOSITY MU W.R.T. EPSEFF SQUARED --
C.... USED IN NEWTON'S METHOD -- DMU = 0. FOR FIRST PASS.
      AMU = 0.5/A
      DMU = 0.
      IF(ISTART.LE.NUMEL)GO TO 291
      AMU = (0.5/A**F1EPS)*EPSEFF**F2EPS
      DMU = (0.25*F2EPS/A**F1EPS)*EPSEFF**(F2EPS-2.)
  291 XMU = AMU*XSJWG
      XDMU2 = DMU/2.*XSJWG
C.... COMPUTE DIVERGENCE TERM
      DIV = DV(1,1) + DV(2,2)
      GAMXY = DV(1,2) + DV(2,1)
C.... COMPUTE INTERNAL FORCES
C.... Q RELATES TO NEWTON'S METHOD ACCOUNTING OF THE DERIVATIVE OF
C.... VISCOSITY.
      J1 = 1
      DO 64 J=1,NEL
      Q(J1) = 2.*DV(1,1)*SHP(1,J) + GAMXY*SHP(2,J)
      P(1,J) = P(1,J) + SHP(1,J)*V(3)*XSJWG - Q(J1)*XMU
      Q(J1+1) = 2.*DV(2,2)*SHP(2,J) + GAMXY*SHP(1,J)
      P(2,J) = P(2,J) + SHP(2,J)*V(3)*XSJWG - Q(J1+1)*XMU
     *                 - XSJWG*RHOG*SHP(3,J)
C.... THE LAST TERM ABOVE IS GRAVITY LOADING
      P(3,J) = P(3,J) + SHPP(3,J)*DIV*XSJWG
      J1 = J1 + NDF
   64 CONTINUE
C.... COMPUTE TANGENT, LOOP OVER COLUMNS OF S
      K1 = 1
      DO 34 K = 1,NEL
      A1 = XMU*SHP(1,K)
      A2 = XMU*SHP(2,K)
      QK1 = Q(K1)*XDMU2
      QK1P1 = Q(K1+1)*XDMU2
C.... LOOP OVER ROWS OF S
      J1 = 1
      DO 33 J = 1,K
      S(J1  ,K1  ) = S(J1  ,K1  ) + SHP(1,J)*(A1+A1)+SHP(2,J)*A2
     * + Q(J1)*QK1
      S(J1  ,K1+1) = S(J1  ,K1+1) + SHP(2,J)*A1 + Q(J1)*QK1P1
      S(J1  ,K1+2) = S(J1  ,K1+2) - SHP(1,J)*SHPP(3,K)*XSJWG
      S(J1+1,K1  ) = S(J1+1,K1  ) + SHP(1,J)*A2 + Q(J1+1)*QK1
      S(J1+1,K1+1) = S(J1+1,K1+1) + SHP(1,J)*A1+SHP(2,J)*(A2+A2)
     * + Q(J1+1)*QK1P1
```

```
      S(J1+1,K1+2) = S(J1+1,K1+2) - SHP(2,J)*SHPP(3,K)*XSJWG
      S(J1+2,K1  ) = S(J1+2,K1  ) - SHPP(3,J)*SHP(1,K)*XSJWG
      S(J1+2,K1+1) = S(J1+2,K1+1) - SHPP(3,J)*SHP(2,K)*XSJWG
33    J1 = J1 + NDF
34    K1 = K1 + NDF
   65 CONTINUE
C.... CHECK IF AT CALVING FACE WHERE ICOLUM=2.
      IF(IX(3).LT.NFACE)GO TO 302
C.... COMPUTE 1-D GAUSS POINTS AND WEIGHTS FROM 2-D FOR CALVING FACE.
      DO 301 L1D=1,LINT1D
      L2D = L1D
      IF(L1D.EQ.3)L2D=9
      SG1D(L1D) = 1.
      TG1D(L1D) = SG(L2D)
  301 WG1D(L1D) = SQRT(WG(L2D))
      ICOLUM = 2
      DELTA = -2.*ATAN(1.)
      GO TO 304
  302 CONTINUE
C.... CHECK IF AT FREE SURFACE WHERE ICOLUM=1.
      IF(MOD(IX(3),NDEEP).NE.1)GO TO 66
C.... COMPUTE 1-D GAUSS POINTS AND WEIGHTS FROM 2-D FOR FREE SURFACE.
      DO 303 L1D=1,LINT1D
      L2D = L1D
      IF(L1D.EQ.3)L2D = 9
      SG1D(L1D) = SG(L2D)
      TG1D(L1D) = 1.
  303 WG1D(L1D) = SQRT(WG(L2D))
      ICOLUM = 1
      DELTA = 2.*ATAN(1.)
C.... COMPUTE WATER + ATMOSPHERIC FORCE AT INTEGRATION POINT ON FACE.
  304 DO 330 L1D=1,LINT1D
C.... CALL 2-D SHAPE FUNCTION WITH LAST ARGUMENT .TRUE.; THIS RETURNS
C.... NATURAL (NOT GLOBAL) DERIVATIVES.
      CALL SHAPE(SG1D(L1D),TG1D(L1D),XL,SHP,XSJ,NDM,NEL,IX,.TRUE.)
C.... COMPUTE GLOBAL COORDINATES CORRESPONDING TO INTEGRATION POINTS.
C.... NATURAL DERIVATIVES FROM COMMON /JACOBI/ ARE
C.... XS(2,ICOLUM)=DY, XS(1,ICOLUM)=DX.
C.... NOTE THAT XS IS THE TRANSPOSE OF THE JACOBIAN.
      YCOORD = 0.
      DO 305 K=1,NEL
  305 YCOORD = YCOORD + XL(2,K)*SHP(3,K)
C.... COMPUTE JACOBIAN = LINE LENGTH TRANSFORMATION
      XSJ = SQRT(XS(1,ICOLUM)**2 + XS(2,ICOLUM)**2)
C.... COMPUTE ANGLE OF FACE, +X TO +NORMAL COORD. ALONG FACE (DY/DX).
      BETA = ATAN2(XS(2,ICOLUM),XS(1,ICOLUM))
C.... COMPUTE ANGLE TO OUTWARD NORMAL TO FACE.
      GAMMA = BETA + DELTA
C.... ADD TO INTERNAL FORCES
      DO 310 J=1,NEL
```

```
      WFORCE = (AMAX1(-YCOORD,0.)*RHOGW+ATMOSP)*XSJ*WG1D(L1D)*SHP(3,J)
      P(1,J) = P(1,J) - WFORCE*COS(GAMMA)
  310 P(2,J) = P(2,J) - WFORCE*SIN(GAMMA)
C.... CHECK IF AT UPPER FREE SURFACE, WHERE ICOLUM=1.
      IF(ICOLUM.NE.1)GO TO 330
C.... ADD HDOT SUBMATRIX TO TANGENT STIFFNESS.
C.... TERMS INVOLVING J2, K2, INCLUDE EMERGENCE CHANGE DUE TO VELOCITY
C.... CHANGE CALCULATED IN THIS ITERATION.  HDOT TERMS ARE MULTIPLIED
C.... BY VERY SMALL ALPHA SO SYMMETRY CAN BE PRESERVED WITHOUT CHANGING
C.... VELOCITY CALCULATION.
      CTNGAM = COS(GAMMA)/SIN(GAMMA)
      DO 315 K=1,3
      K1 = MID(K)*NDF
      K2 = ITOP(K)*NDF
      SHK = ALPHA*SHP(3,ITOP(K))*XSJ*WG1D(L1D)
      DO 315 J=1,3
      J1 = MID(J)*NDF
      J2 = ITOP(J)*NDF
      SHJSHK = SHP(3,ITOP(J))*SHK
      S(J2-2,K1) = S(J2-2,K1) - CTNGAM*SHJSHK
      S(J2-1,K1) = S(J2-1,K1) - SHJSHK
      S(J1,K2-1) = S(J1,K2-1) - SHJSHK
      S(J1,K2-2) = S(J1,K2-2) - CTNGAM*SHJSHK
  315 S(J1,K1) = S(J1,K1) + SHJSHK
C.... COMPUTE BALANCE AND OLD HDOT (SO ITERATION YIELDS DELTA HDOT)
C.... AT INTEGRATION POINT.
      BALANC = 0.
      HDOT = 0.
      DO 316 J=1,3
      HDOT = HDOT + UL(3,MID(J))*SHP(3,ITOP(J))
      CALL VALUES(BALNOD,M(N10),3,IX(MID(J))+2,NDF)
  316 BALANC = BALANC + BALNOD*SHP(3,ITOP(J))
C.... COMPUTE SURFACE VELOCITY AT INTEGRATION POINT
      DO 320 I=1,2
      V(I) = 0.
      DO 320 J=1,3
  320 V(I) = V(I) + SHP(3,ITOP(J))*UL(I,ITOP(J))
C.... COMPUTE EMERGENCE
      EMERG = (V(1)*COS(GAMMA) + V(2)*SIN(GAMMA))/SIN(GAMMA)
C.... ADD TO RIGHT HAND SIDE
      DO 325 J=1,3
  325 P(3,MID(J)) = P(3,MID(J))
     * + ALPHA*(EMERG + BALANC - HDOT)*SHP(3,ITOP(J))*XSJ*WG1D(L1D)
  330 CONTINUE
      IF(ICOLUM.EQ.2)GO TO 302
C.... FORM LOWER PART OF TANGENT STIFFNESS MATRIX BY SYMMETRY.
   66 NL = NDF*NEL
      DO 68 J=2,NL
      DO 68 K=1,J
   68 S(J,K) = S(K,J)
```

```
      RETURN
C.... COMPUTE STRESSES AND VELOCITY GRADIENTS
4     RETURN
C.... COMPUTE MASS MATRIX
5     RETURN
C.... FORMATS
1000  FORMAT(10F10.0)
2000  FORMAT(5X,32HTWO DIMENSIONAL ICE FLOW ELEMENT//
     1  10X,12HVISCOS. A  =,E12.5/
     2  10X,12HVISCOS. N  =,E12.5/
     3  10X,12HCALVE NODE =,E12.5/
     4  10X,12HNODES DEEP =,E12.5/)
      END
      SUBROUTINE PLT(ID,X,B,F,T,D,TIME,NDM,NDF)
C
C.... PLOT DISPLACEMENT VECTORS B AT NODAL COORDINATES X
C
      LOGICAL PCOMP
      COMMON /PRLOD/PROP
      COMMON /CDATA/ O,HEAD(20),NUMNP,NUMEL,NUMMAT,NEN,NEQ,IPR
      COMMON /NORIG/ NUMEXT
      DIMENSION X(NDM,1),B(1),UL(6),ID(NDF,1),F(NDF,1),WD(2),D(1),T(1)
      DATA BL/4HBLAN/,DOTFAC/0.3/,TICFAC/2./,NWD/2/
      DATA WD/4HSCAL,4HEND /,FIRST/0./
C.... FUNCTIONS TO APPLY ORIGIN SHIFT, SCALE, AND ROTATION
      X2(XP,YP) = (SHIFT + COSINE*(XP-XMIN) - SINE*(YP-YMIN))/SCALE
      Y2(XP,YP) = (SINE*(XP-XMIN) + COSINE*(YP-YMIN))/SCALE
C.... FIRST=0 MEANS READ IN NEW SCALES AS THIS IS THE FIRST
C.... TIME THROUGH THIS PLOT MACRO
      IF(ABS(FIRST).GT..5)GO TO 2
      FIRST = 1.
   10 READ(5,5000)CC
      DO 20 J=1,NWD
   20 IF(PCOMP(CC,WD(J)))GO TO 30
      GO TO 10
   30 GO TO (1,2),J
C.... READ SCALE INFORMATION
    1 READ(5,5001)XMIN,XMAX,YMIN,YMAX,SCALE,VSCALE,AHLEN,ROT
      DOTLEN = DOTFAC*AHLEN
      BLOWUP = SCALE/VSCALE
      ANGLE = 0.
      COSINE = 1.
      SINE = 0.
      SHIFT = 0.
      IF(ABS(ROT-90.).GT..5)GO TO 15
      COSINE = 0.
      SINE = 1.
      ANGLE = 90.
      SHIFT = YMAX - YMIN
   15 CONTINUE
```

```
      CALL PLOTS(0,0,99)
      GO TO 10
C.... PLOT DISPLACEMENT VECTORS
    2 CALL NEWPEN(2)
      DO 202 N=1,NUMNP
      IF(PCOMP(X(1,N),BL))GO TO 202
      DO 200 I=1,NDF
      UL(I) = F(I,N)*PROP*BLOWUP
      K = IABS(ID(I,N))
  200 IF(K.GT.0)UL(I)=B(K)*BLOWUP
      XTAIL = X(1,N)
      YTAIL = X(2,N)
      IF(XTAIL.LT.XMIN.OR.XTAIL.GT.XMAX.OR.YTAIL.LT.YMIN.OR.
     * YTAIL.GT.YMAX)GO TO 202
      CALL SYMBOL(X2(XTAIL,YTAIL),Y2(XTAIL,YTAIL),DOTLEN,1,ANGLE,-1)
      DV = SQRT(UL(1)**2 + UL(2)**2)/SCALE
      IF(DV.LT.DOTLEN)GO TO 202
      AHLEN2 = AMIN1(AHLEN,DV)
      XTIP = XTAIL + UL(1)
      YTIP = YTAIL + UL(2)
      IF(XTIP.LT.XMIN.OR.XTIP.GT.XMAX.OR.YTIP.LT.YMIN.OR.
     * YTIP.GT.YMAX)GO TO 202
      CALL AROHD(X2(XTAIL,YTAIL),Y2(XTAIL,YTAIL),X2(XTIP,YTIP),
     1 Y2(XTIP,YTIP),AHLEN2,0.,14)
  202 CONTINUE
      NDEEP = D(4) + 0.1
      CALL NEWPEN(1)
C.... PLOT BED, SMOOTHED.
      TICK = TICFAC*AHLEN*SCALE
      XR = X2(T(1),T(2))
      YR = Y2(T(1),T(2))
      CALL PLOT(XR,YR,3)
      CALL SMOOT(XR,YR,0)
      NDEEP1 = NDEEP + 1
      NUMEXM = NUMEXT - NDEEP
      NUMNPM = NUMNP - NDEEP
      DO 330 I=NDEEP1,NUMEXM,NDEEP
      XR = X2(T(I),T(I+1))
      YR = Y2(T(I),T(I+1))
      CALL SMOOT(XR,YR,-2)
      XP = T(I-NDEEP)
      YP = T(I-NDEEP+1) - TICK
      XR = X2(XP,YP)
      YR = Y2(XP,YP)
      CALL SMOOT(XR,YR,2)
      YP = YP + TICK
      XR = X2(XP,YP)
      YR = Y2(XP,YP)
  330 CALL SMOOT(XR,YR,2)
      XP = T(NUMEXM+1)
```

```
      YP = T(NUMEXM+2)
      XR = X2(XP,YP)
      YR = Y2(XP,YP)
      CALL SMOOT(XR,YR,-24)
      YP = YP - TICK
      XR = X2(XP,YP)
      YR = Y2(XP,YP)
      CALL PLOT(XR,YR,2)
      YP = YP + TICK
      XR = X2(XP,YP)
      YR = Y2(XP,YP)
      CALL PLOT(XR,YR,2)
      K = NUMEXM - NDEEP + 1
      XP = T(K)
      YP = T(K+1)
      XR = X2(XP,YP)
      YR = Y2(XP,YP)
      CALL PLOT(XR,YR,3)
      YP = YP - TICK
      XR = X2(XP,YP)
      YR = Y2(XP,YP)
      CALL PLOT(XR,YR,2)
      YP = YP + TICK
      XR = X2(XP,YP)
      YR = Y2(XP,YP)
      CALL PLOT(XR,YR,2)
C.... PLOT SURFACE, SMOOTHED.
      XR = X2(X(1,1),X(2,1))
      YR = Y2(X(1,1),X(2,1))
      CALL PLOT(XR,YR,3)
      CALL SMOOT(XR,YR,0)
      DO 340 I=NDEEP1,NUMNPM,NDEEP
      XR = X2(X(1,I),X(2,I))
      YR = Y2(X(1,I),X(2,1))
  340 CALL SMOOT (XR,YR,-2)
      XR = X2(X(1,NUMNPM+1),X(2,NUMNPM+1))
      YR = Y2(X(1,NUMNPM+1),X(2,NUMNPM+1))
      CALL SMOOT(XR,YR,-24)
C.... PLOT CALVING FACE
      XP = X(1,NUMNPM+1)
      YP = X(2,NUMNPM+1)
      XR = X2(XP,YP)
      YR = Y2(XP,YP)
      CALL PLOT(XR,YR,3)
      XP = X(1,NUMNP)
      YP = X(2,NUMNP)
      XR = X2(XP,YP)
      YR = Y2(XP,YP)
      CALL PLOT(XR,YR,2)
C.... PLOT SEALEVEL.
```

```
      XP = X(1,NUMEXT)
      YP = 0.
      XR = X2(XP,YP)
      YR = Y2(XP,YP)
      CALL PLOT(XR,YR,3)
      XP = X(1,1)
      YP = 0.
      XR = X2(XP,YP)
      YR = Y2(XP,YP)
      CALL PLOT(XR,YR,2)
C.... PLOT TIME.
      K = NUMNPM/2 - NDEEP + 1
      XP = T(K)
      YP = T(K+1) - TICK*3.
      XR = X2(XP,YP)
      YR = Y2(XP,YP)
      CALL NUMBER(XR,YR,3.*AHLEN,TIME+1900.,ANGLE,3)
      CALL PLOT(X2(XMAX,YMIN),0.,-3)
      RETURN
 5000 FORMAT(A4)
 5001 FORMAT(8F10.0)
      END
      SUBROUTINE REST(ID,X,B,F,T,D,NDM,NDF,V1)
C
C.... READ NODE NUMBER N, COORDINATES X, AND DISPLACEMENTS B
C.... FROM RESTART TAPE15
C
      COMMON /CDATA/ O,HEAD(20),NUMNP,NUMEL,NUMMAT,NEN,NEQ,IPR
      COMMON /RSTART/ ISTART
      COMMON /NORIG/ NUMEXT
      DIMENSION X(NDM,1),B(1),UL(6),ID(NDF,1),XR(3),T(1),F(NDF,1),D(1)
      REWIND 15
C.... SHIFT "FIXED" BED COORDINATES, BDOT, AND "A" TO TEMPERATURE ARRAY
      NDEEP = D(4) + 0.1
      DO 5 N=NDEEP,NUMNP,NDEEP
      DO 4 I=1,NDM
    4 T(N-NDEEP+I) = X(I,N)
      T(N-NDEEP+4) = F(3,N-NDEEP+4)
    5 T(N-1) = F(3,N-1)
C.... SET TOTAL NUMBER OF NODES IN EXTENDED GRID, BEFORE
C.... NUMNP IS REDUCED IN SUBROUTINE ELMT04.
      NUMEXT = NUMNP
      NVBLS = NDM + NDF
   10 IF(NVBLS.LE.4)READ(15,15000)N,(XR(I),I=1,NDM),(UL(I),I=1,NDF)
      IF(NVBLS.GT.4)READ(15,15001)N,(XR(I),I=1,NDM),(UL(I),I=1,NDF)
C.... CHECK FOR BLANK EOF INDICATOR
      IF(N.LE.0.OR.N.GT.NUMNP)GO TO 103
C.... SKIP REDEFINITION OF CURRENT COORDINATES, BDOT, ABED UNLESS
C.... V1 IS POSITIVE.
      IF(V1.LE.0.)GO TO 95
```

```
      DO 90 I=1,NDM
   90 X(I,N) = XR(I)
      IF(MOD(N,NDEEP).EQ.4.OR.MOD(N,NDEEP).EQ.NDEEP-1)F(3,N) = UL(3)
   95 DO 100 I=1,NDF
      K = IABS(ID(I,N))
  100 IF(K.GT.0)B(K)=UL(I)
      GO TO 10
  103 REWIND 15
C.... SET FLAG SO STRAIN-DEPENDENT VISCOSITY USED
      ISTART = NUMEL + 1
      RETURN
15000 FORMAT(I16,4(1PE16.9))
15001 FORMAT(I16,4(1PE16.9)/5(1PE16.9))
      END
      SUBROUTINE SAVE(ID,X,B,F,NDM,NDF)
C
C.... SAVE NODE NUMBER N, COORDINATES X, AND DISPLACEMENTS B
C.... ON TAPE16.
C
      LOGICAL PCOMP
      COMMON /PRLOD/PROP
      COMMON /CDATA/ O,HEAD(20),NUMNP,NUMEL,NUMMAT,NEN,NEQ,IPR
      DIMENSION X(NDM,1),B(1),UL(6),ID(NDF,1),F(NDF,1)
      DATA BL/4HBLAN/
      REWIND 16
      NVBLS = NDM + NDF
      DO 102 N=1,NUMNP
      IF(PCOMP(X(1,N),BL))GO TO 102
      DO 100 I=1,NDF
      UL(I) = F(I,N)*PROP
      K = IABS(ID(I,N))
  100 IF(K.GT.0)UL(I) = B(K)
      IF(NVBLS.LE.4)WRITE(16,16000)N,(X(I,N),I=1,NDM),(UL(I),I=1,NDF)
      IF(NVBLS.GT.4)WRITE(16,16002)N,(X(I,N),I=1,NDM),(UL(I),I=1,NDF)
  102 CONTINUE
C.... WRITE END OF DATA INDICATOR -- BLANK CARD
      WRITE(16,16001)
      IF(NVBLS.GT.4)WRITE(16,16001)
16001 FORMAT(1H )
      END FILE 16
      REWIND 16
      RETURN
16000 FORMAT(I16,4(1PE16.9))
16002 FORMAT(I16,4(1PE16.9)/5(1PE16.9))
      END
      SUBROUTINE UPNODE(D,X,B,DT,F,T,TIME,ID,NDM,NDF)
C.... UPDATE MESH FOR 2-D DYNAMIC GLACIER PROBLEM
      COMMON /CDATA/O,HEAD(20),NUMNP,NUMEL,NUMMAT,NEN,NEQ,IPR
      COMMON /NORIG/ NUMEXT
      DIMENSION D(1),X(NDM,1),B(1),ID(NDF,1),FACT(3),T(1),F(NDF,1)
```

```
      DIMENSION DKNIK(40),IXS(3),IXB(3),IX(9),XDUM(1,5),SHP(3,9)
      DIMENSION ISURF(3)
      DATA FACT/0.4,0.8,0.9/
      DATA RHOW/1./,RHOI/0.9/,ACONST/1165625./,BCONST/0.550989/
      DATA CCONST/-2.23920/,IX/1,1,0,0,1,0,0,0,0/,TT/-1./
      DATA XDUM/5*0./,ISURF/1,2,5/,SHP/27*0./
      DATA DKNIK/24.27, 23.69, 25.38, 28.15, 29.78, 29.36, 27.44,
     * 21.51, 22.95, 23.86, 25.43, 26.12, 44.59, 84.95, 99.69, 143.10,
     * 220.03, 317.97, 426.72, 479.86, 542.15, 607.59, 656.56, 720.51,
     * 805.18, 705.21, 476.12, 356.48, 372.13, 257.24, 157.36, 135.86,
     * 83.29, 61.79, 62.53, 44.41, 32.71, 30.47, 27.13, 25.63/
      NDEEP = D(4) + 0.1
      NDEEP1 = NDEEP + 1
      NDPM4 = NDEEP - 4
      NDPM5 = NDEEP - 5
C.... FIRST DETERMINE UNSUPPORTED HEIGHT HU.
C.... DO BEFORE PROJECTING SURFACE TO (TIME+DT).
      HG = X(2,NUMNP-NDEEP+1)
      HW = AMAX1(0.,-X(2,NUMNP))
      HU = HG - (RHOW/RHOI - 1.)*HW
C.... LOCATE ADJUSTED SURFACE ALTITUDE.
      DO 10 I=NDEEP1,NUMNP,NDEEP
      K = IABS(ID(3,I+1))
   10 X(2,I) = X(2,I) + B(K)*DT
C.... DETERMINE DELX.
C.... FIRST DETERMINE V.
      K = IABS(ID(1,NUMNP-NDEEP+1))
      V = B(K)
C.... NEXT DETERMINE DISCHARGE "DISCH" FOR THE TIME INTERVAL "TIME" TO
C.... "TIME + DT".
C.... NEXT DETERMINE DISCHARGE FROM THE SAME INTERVAL.
      I = AMOD(TIME,1.0)/DT + 1.1
      DISCH = DKNIK(I)
C.... DETERMINE CALVING SPEED USING CALVING LAW.
      VC = ACONST*DISCH**BCONST*HU**CCONST
C.... DETERMINE DELX
      DELX = (V - VC)*DT
C.... FIND ORIGINAL TOTAL GLACIER LENGTH.
      XTORIG = T(NUMNP-NDEEP+1)
      X0ORIG = T(1)
      ORIGL = XTORIG - X0ORIG
C.... FIND THE BED LENGTH.
      XBED = T(NUMEXT-NDEEP+1)
      BEDLEN = XBED - X0ORIG
C.... FIND NEW CURRENT TOTAL LENGTH.
      CURRL = X(1,NUMNP-NDEEP+1) + DELX
C.... INSURE THAT TERMINUS REMAINS WITHIN THE BED LENGTH.
      CURRL = AMIN1(BEDLEN,CURRL)
      CURRL = AMAX1(CURRL,0.)
C.... FIND NEW COLUMN X-COORDINATES -- PROPORTIONED
```

```
C.... AS IN ORIGINAL MESH.
C.... THE FIRST COLUMN REMAINS FIXED -- START WITH THE SECOND.
      IXS(1) = 1
      IXS(3) = 1 + NDEEP
      IXS(2) = 1 + 2*NDEEP
      IXB(1) = 1
      IXB(3) = 1 + NDEEP
      IXB(2) = 1 + 2*NDEEP
      DO 300 I=NDEEP1,NUMNP,NDEEP
C.... FIND ORIGINAL X-COORDINATE OF COLUMN.
      XIORIG = T(I)
C.... FIND PROPORTIONATE NEW X-COORDINATE.
      XSURF = (XIORIG - X0ORIG)/ORIGL*CURRL + X0ORIG
C.... REPLACE X-COORDINATE OF ALL NODES IN THIS COLUMN EXCEPT
C.... THE SURFACE NODE.
      DO 40 J=2,NDEEP
   40 X(1,I+J-1) = XSURF
C.... DETERMINE CORRECT ELEMENT FOR SURFACE ALTITUDE INTERPOLATION.
   50 IF(IXS(2).GE.NUMNP-NDEEP+1)GO TO 60
      IF(XSURF.LE.X(1,IXS(2)))GO TO 60
      DO 55 K=1,3
   55 IXS(K) = IXS(K) + NDEEP
      GO TO 50
   60 CONTINUE
C.... INTERPOLATE SURFACE.
C.... FIRST FIND CORRESPONDING NATURAL COORDINATES.
      IF(XSURF.GE.X(1,IXS(3)))
     * SS = (XSURF - X(1,IXS(3))) / (X(1,IXS(2)) - X(1,IXS(3)))
      IF(XSURF.LT.X(1,IXS(3)))
     * SS = (XSURF - X(1,IXS(3))) / (X(1,IXS(3)) - X(1,IXS(1)))
      CALL SHAPE(SS,TT,XDUM,SHP,XSJ,1,5,IX,.TRUE.)
      YSURF = 0.
      DO 70 K=1,3
   70 YSURF = YSURF + SHP(3,ISURF(K))*X(2,IXS(K))
C.... STORE INTERPOLATED NEW CURRENT Y-COORDINATE OF THE SURFACE NODE
C.... IN PLACE OF THE Y-COORDINATE OF THE SECOND NODE OF THE COLUMN,
C.... TEMPORARILY.
      X(2,I+1) = YSURF
C.... DETERMINE CORRECT ELEMENT FOR BED INTERPOLATION.
  250 IF(IXB(2).GE.NUMEXT-NDEEP+1)GO TO 260
      IF(XSURF.LE.T(IXB(2)))GO TO 260
      DO 255 K=1,3
  255 IXB(K) = IXB(K) + NDEEP
      GO TO 250
  260 CONTINUE
C.... INTERPOLATE BED.
C.... FIRST FIND CORRESPONDING NATURAL COORDINATES.
      IF(XSURF.GE.T(IXB(3)))
     * SS = (XSURF - T(IXB(3))) / (T(IXB(2)) - T(IXB(3)))
      IF(XSURF.LT.T(IXB(3)))
```

```
     * SS = (XSURF - T(IXB(3))) / (T(IXB(3)) - T(IXB(1)))
      CALL SHAPE(SS,TT,XDUM,SHP,XSJ,1,5,IX,.TRUE.)
      YBED = 0.
      BCURR = 0.
      ACURR = 0.
      DO 270 K=1,3
      YBED = YBED + SHP(3,ISURF(K))*T(IXB(K)+1)
      BCURR = BCURR + SHP(3,ISURF(K))*T(IXB(K)+3)
  270 ACURR = ACURR + SHP(3,ISURF(K))*T(IXB(K)+NDEEP-2)
C.... REPLACE OLD "CURRENT" BED ALTITUDE, BDOT, AND ABED WITH
C.... NEW ONES (CORRESPONDING TO XSURF NOW).
      X(2,I+NDEEP-1) = YBED
      X(2,I+NDEEP-2) = YBED + 5.
      X(2,I+NDEEP-3) = YBED + 10.
      X(2,I+NDEEP-4) = YBED + 10.
      F(3,I+3) = BCURR
      F(3,I+NDEEP-2) = ACURR
  300 CONTINUE
C.... REPLACE COLUMN COORDINATES WITH NEW INTERPOLATED ONES.
      DO 310 I=NDEEP1,NUMNP,NDEEP
      X(1,I) = X(1,I+1)
      X(2,I) = X(2,I+1)
      DELTA = X(2,I) - X(2,I+NDPM4)
      DO 310 K=1,NDPM5
  310 X(2,I+K) = X(2,I) - DELTA*FACT(K)
      RETURN
      END
      SUBROUTINE VALUES(VALUE,F,I,J,NDF)
C.....LOCATE BOUNDARY VALUES FOR SUBROUTINE ELMT04 IN COMMON M.
      DIMENSION F(NDF,1)
      VALUE = F(I,J)
      RETURN
      END
```

APPENDIX D: MODIFICATIONS OF THE GENERAL PURPOSE FINITE ELEMENT PROGRAM

Included below are modifications to the general purpose finite element program as it is listed in Zienkiewicz (1977). The subroutine FREEFM is from R. L. Taylor (written commun., 1980), as received from him in the card deck of the general purpose program. It is included here for completeness, so that the reader may refer to the program listed in Zienkiewicz. Some single line modifications listed below are also from Taylor; these are indicated either by explicit statement to that effect or by identification of the form CR 4/80C in columns 73 through 80 of the card. Changes with blanks in these columns are attributable to the author. Enough lines of the general purpose subroutines of Zienkiewicz (1977) are given so that the reader can locate the changes. The inclusion of modifications by Taylor is meant only to specify the exact form of the program used in the modeling. It is strongly recommended that the interested reader obtain the latest version of the program directly from Taylor.

```
C**** COMMENT OUT CDC PROGRAM CARD NOT REQUIRED ON IBM.
C     PROGRAM FEAPB(INPUT=101,OUTPUT=101,TAPE4=INPUT,TAPE5=101,
C    1                 TAPE6=OUTPUT)
C     MASTER MINIFEM                                                    IFEM  1C
C.... SET PROGRAM CAPACITY * MAX MUST AGREE WITH DIMENSION OF M         IFEM  2C
C**** CHANGE PROGRAM CAPACITY FOR PROBLEM AT HAND.
      COMMON M(20000)
      COMMON/PSIZE/ MAX                                                 IFEM  4C
C**** CHANGE MAX TO AGREE WITH DIMENSION OF M.
      MAX = 20000
      CALL FREEFM(M,M(11),M(21),M(31),M(111))                          CR 4/80C
      CALL PCONTR                                                       IFEM  6C
      STOP                                                              IFEM  7C
      END                                                               IFEM  8C
      SUBROUTINE FREEFM(IS,IE,IT,IN,IWD)                                 FRE  01
C.... FREE FORMATED READ * * CONVERTED TO FIELD WIDTHS OF TEN (10)       FRE  02
C.... CARDS WHICH BEGIN WITH A STAR (*) ARE ALPHANUMERIC                 FRE  03
C.... A CARD WHICH BEGINS WITH TWO STARS (**) WILL TERMINATE INPUT       FRE  04
      LOGICAL BL,FL                                                      FRE  05
      DIMENSION IS(10),IE(10),IT(10),IN(80),IWD(80)                      FRE  06
C**** CHANGE 10H TO 01H TO ACCOMMODATE IBM (STILL OK ON CDC).
C     DATA NDIG,IBL/10,10H          /,ISL/1H//
      DATA NDIG,IBL/10,01H          /,ISL/1H//
      DATA IA,IC,IR,IZ/1HA,1H,,1HR,1HZ/,IST/1H*/                         FRE  08
      REWIND 5                                                           FRE  09
      WRITE(6,2000)                                                      FRE  10
      FL = .TRUE.                                                        FRE  11
10    READ(4,1000) IN                                                    FRE  12
      ILM = 1                                                            FRE  13
      DO 20 I = 1,80                                                     FRE  14
      IF(IN(I).NE.IBL) ILM = I +1                                        FRE  15
20    CONTINUE                                                           FRE  16
      IL = 1                                                             FRE  17
```

```
30    JL = IL + 1
      DO 35 I = 1,80
35    IWD(I) = IBL
      IF(IN(IL).NE.IST) GO TO 50
      IF(IN(JL).EQ.IST) GO TO 160
      DO 40 I = JL,ILM
      IF(IN(I).EQ.ISL) GO TO 45
40    IWD(I-IL) = IN(I)
      IL = ILM + 1
      GO TO 150
45    IL = I + 1
      GO TO 150
50    DO 100 I = 1,10
      IS(I) = 0
100   IT(I) = IR
      BL = .TRUE.
      K = 1
105   J = IN(IL)
      IF(J.EQ.IBL.AND.BL) GO TO 120
      IF(J.EQ.IBL.OR.J.EQ.IC.OR.J.EQ.ISL) GO TO 110
      IF(.NOT.BL) GO TO 120
      BL = .FALSE.
      IS(K) = IL
      IF(J.GE.IA.AND.J.LE.IZ) IT(K) = IA
      GO TO 120
110   BL = .TRUE.
      IE(K) = IL - 1
      K = K + 1
120   IL = IL + 1
      IF(K.GT.8.OR.J.EQ.ISL) GO TO 125
      IF(IL.LE.ILM) GO TO 105
125   K = K - 1
      IF(K.EQ.0) GO TO 150
      KL = 1
      KR = NDIG
      DO 145 I = 1,K
      JL = IS(I)
      IF(JL.EQ.0) GO TO 144
      JR = IE(I)
      IF(JR-JL.GT.9) GO TO 200
      II = KR - JR
      IF(IT(I).EQ.IA) II = KL - JL
      DO 140 J = JL,JR
140   IWD(J+II) = IN(J)
144   KL = KR + 1
145   KR = KR + NDIG
150   WRITE(6,2001) IWD
      WRITE(5,1000) IWD
      IF(IL.LE.ILM.OR.IN(IL-1).EQ.ISL) GO TO 30
      GO TO 10
160   REWIND 5
      WRITE(6,2002)
      IF(FL) RETURN
      STOP
```

```
200   WRITE(6,3000) IN                                                  FRE  72
      FL = .FALSE.                                                      FRE  73
      GO TO 10                                                          FRE  74
1000  FORMAT(80A1)                                                      FRE  75
2000  FORMAT(1H1,54HFIXED FORMAT FORM OF DATA CARDS FROM FREE FORMAT INP FRE  76
     1UT        ///1X)                                                  FRE  77
2001  FORMAT(5X,80A1)                                                   FRE  78
2002  FORMAT(1H0,17HEND OF DATA INPUT ///1X)                            FRE  79
3000  FORMAT(5X,48HMORE THAN TEN CHARACTERS/DIGITS ON THE DATA CARD//   FRE  80
     1          10X,80A1)                                               FRE  81
      END                                                               FRE  82
      SUBROUTINE PCONTR                                                 PCO  1C
                                 .
                                 .
                                 .
      COMMON M(1)                                                       PCO 14C
C**** ADD THE FOLLOWING COMMON SO VARIABLES IN M CAN BE FOUND.
      COMMON /POINTR/ N0,N1,N2,N3,N4,N5,N6,N7,N8,N9,N10,N11,N12,N13,
     * N14,NE
      DIMENSION TITL(20),WD(3)                                          PCO 15C
                                 .
                                 .
                                 .
      CALL SETMEM(NE)                                                   PCO 59C
      CALL PZERO(M(N13),NEQ)                                          CR10/77C
      GO TO 1                                                           PCO 60C
                                 .
                                 .
                                 .
C**** FROM R.L. TAYLOR -- NEW INPUT FORMAT.

1001  FORMAT(8I10)                                                      PCO 68C
                                 .
                                 .
                                 .
      END                                                               PCO 74C
      BLOCK DATA                                                            1C
                                 .
                                 .
                                 .
C**** SINGLE PRECISION IS SUFFICIENT.
      DATA O/1H0/,IPR/1/
      END                                                                   9C
      SUBROUTINE GENVEC(NDM,X,CD,PRT,ERR)                               GEN  1C
                                 .
                                 .
                                 .
C**** FROM R.L. TAYLOR -- NEW INPUT FORMAT.
1000  FORMAT(2I10,6F10.0)                                               GEN 38C
                                 .
                                 .
                                 .
      END                                                               GEN 44C
```

```
      SUBROUTINE PMESH(IDL,IE,D,ID,X,IX,F,T,NDF,NDM,NEN1,III)              PME   1C
                                     .
                                     .
                                     .
C**** REMOVE DATA SPECIFICATION FOR UNUSED VARIABLE VA.
      DATA WD/4HCOOR,4HELEM,4HMATE,4HBOUN,4HFORC,4HTEMP,4HEND ,4HPRIN,    PME  11C
C    1  4HNOPR,4HPAGE/,BL/4HBLAN/,VA/4H VAL,2HUE/,LIST/10/,PRT/.TRUE./
     1  4HNOPR,4HPAGE/,BL/4HBLAN/,                LIST/10/,PRT/.TRUE./
C.... INITIALIZE ARRAYS                                                    PME  13C
                                     .
                                     .
                                     .
      ID(I,N) = 0                                                          PME  18C
      X(1,N) = BL                                                         CR 5/79C
100   F(I,N) = 0.0                                                         PME  19C
                                     .
                                     .
                                     .
C**** FROM R.L. TAYLOR -- OMIT XHED FROM VARIABLES READ.
      READ(5,1002) MA,IEL                                                  PME  60C
C**** FROM R.L. TAYLOR -- OMIT XHED FROM VARIABLES WRITTEN.
      WRITE(6,2003)MA,IEL                                                  PME  61C
      IE(MA) = IEL                                                         PME  62C
300    CALL ELMLIB(D(1,MA),IDL,X,IX,IDL,IDL,IDL,NDF,NDM,NDF,1)            CR 8/77C

      GO TO 10                                                             PME  64C
                                     .
                                     .
                                     .
      DO 53 I = 1,NDF                                                      PME  80C
C**** GENERATION FOR TIED DEGREES OF FREEDOM.
      IF(ID(I,L-LG).GT.1)ID(I,L)=ID(I,L-LG) + LG
53    IF(ID(I,L-LG).LT.0) ID(I,L) = -1                                     PME  81C
                                     .
                                     .
                                     .
C**** FROM R.L. TAYLOR -- NEW INPUT FORMAT.
1001  FORMAT(8I10)                                                         PME105C
C**** FROM R.L. TAYLOR -- NEW INPUT FORMAT.
1002  FORMAT(2I10)                                                         PME106C
                                     .
                                     .
                                     .
      END                                                                  PME117C
      SUBROUTINE PMACR (UL,XL,TL,LD,P,S,IE,D,ID,X,IX,F,T,JDIAG,B,DR,CT    PMA   1C
     1,NDF,NDM,NEN1,NST,NEND)                                              PMA   2C
                                     .
                                     .
                                     .
      COMMON /TDATA/ TIME,DT,C1,C2,C3,C4,C5                                PMA  14C
C**** INCREASE THE NUMBER OF MACROS.
      DIMENSION WD(27),CT(4,1),CTL(4),LVS(9),LVE(9),JDIAG(1),
     1   UL(1),XL(1),TL(1),LD(1),P(1),S(1),IE(1),D(1),ID(1),X(1),         PMA  16C
     2   IX(1),F(1),T(1),B(1),DR(1)                                        PMA  17C
```

```
C**** INCREASE THE NUMBER OF MACROS.
      DATA WD/4HTOL ,4HDT  ,4HSTRE,4HDISP,4HTANG,4HFORM,4HLOOP,4HNEXT,    PMA 18C
     1        4HPROP,4HDATA,4HTIME,4HCONV,4HSOLV,4HLMAS,4HCMAS,4HMESH,    PMA 19C
     2        4HEIGE,4HEXCD,4HUTAN,4HREAC,4HCHEC,4HSAVE,4HREST,4HPLOT,
     3        4HUTAF,4HTANF,4HUPNO/
      DATA NWD/27/,ENDM/4HEND /,NV,NC/1,1/
C.... SET INITIAL VALUES OF PARAMETERS                                     PMA 22C
                               .
                               .
                               .
      IF(L.NE.1.AND.L.NE.LL)                                               PMA 85C
     1WRITE(6,2010) I,(CT(K,L),K = 1,4)                                    PMA 86C
C**** INCREASE THE NUMBER OF MACROS.
      GO TO (1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,
     * 22,23,24,25,26,27),J
C.... SET SOLUTION TOLERANCE                                               PMA 88C
                               .
                               .
                               .
      CALL PFORM(UL,XL,TL,LD,P,S,IE,D,ID,X,IX,F,T,JDIAG,DR,DR,DR,          PMA121C
     2   NDF,NDM,NEN1,NST,6,B,M(NV),.FALSE.,.TRUE.,.FALSE.,.FALSE.)        PMA122C

C**** ADD STATEMENT NUMBER 60.
   60 BFR = .TRUE.
      RN = 0.                                                              PMA124C
                               .
                               .
                               .
21    CALL PFORM(UL,XL,TL,LD,P,S,IE,D,ID,X,IX,F,T,JDIAG,DR,DR,DR,          PMA229C
     1   NDF,NDM,NEN1,NST,2,B,F,.FALSE.,.FALSE.,.FALSE.,.FALSE.)           PMA230C
C**** ADD MACROS.
      GO TO 330
C.... SAVE NODE NUMBER N, COORDINATES X, AND DISPLACEMENTS B
C.... ON TAPE16.
   22 CALL SAVE(ID,X,B,F,NDM,NDF)
      GO TO 330
C.... READ NODE NUMBER N, COORDINATES X (IF CT(3,L) = 1.),
C.... AND DISPLACEMENTS B FROM RESTART TAPE15.
C.... IF CT(3,L) = 0., READ ONLY DISPLACEMENTS.
   23 CALL REST(ID,X,B,F,T,D,NDM,NDF,CT(3,L))
      GO TO 330
C.... PLOT
   24 CALL PLT(ID,X,B,F,T,D,TIME,NDM,NDF)
      GO TO 330
C.... FORM TANGENT STIFFNESS AND OUT OF BALANCE FORCES, ONE CALL
C.... PER ELEMENT.
   25 IF(CFL)CALL PSETM(NC,NE,JDIAG(NEQ)*IPR,CFL)
      CALL PZERO(M(NC),JDIAG(NEQ))
      CFR = .TRUE.
   26 IF(J.EQ.26)CFR = .FALSE.
      IF(GFL)CALL PSETM(NA,NE,JDIAG(NEQ)*IPR,GFL)
      IF(NPLD.GT.0)PROP = PROPLD(TIME,0)
      CALL PZERO(M(NA),JDIAG(NEQ))
      CALL PLOAD(ID,F,DR,NNEQ,PROP)
```

```
      CALL PFORM(UL,XL,TL,LD,P,S,IE,D,ID,X,IX,F,T,JDIAG,DR,M(NA),M(NC),
     2   NDF,NDM,NEN1,NST,3,B,M(NV),.TRUE.,.TRUE.,CFR,.FALSE.)
      AFR = .TRUE.
      GO TO 60
C.... UPDATE MESH
   27 CALL UPNODE(D,X,B,DT,F,T,TIME,ID,NDM,NDF)
330   L = L + 1                                                     PMA231C
                              .
                              .
                              .
C**** FROM R.L. TAYLOR -- NEW INPUT FORMAT.
1000  FORMAT(A4,6X,A4,6X,2F10.0)                                    PMA243C
                              .
                              .
                              .
      END                                                           PMA264C

      SUBROUTINE ACTCOL(A,B,JDIAG,NEQ,AFAC,BACK)                    ACT   1C
                              .
                              .
                              .
      ID = JDIAG(K+I)                                               ACT  33C
C**** GIVE ERROR MESSAGE FOR DIVISION BY 0.
      IF(A(ID).NE.0.0) GO TO 310
      INDEX = K + I
      GO TO 1100
  310 D = A(I)
      A(I) = A(I)/A(ID)                                             ACT  36C
                              .
                              .
                              .
      ID = JDIAG(I)                                                 ACT  45C
C**** GIVE ERROR MESSAGE FOR DIVISION BY 0.
      IF(A(ID).NE.0.0)GO TO 610
      INDEX = I
      GO TO 1100
  610 B(I) = B(I)/A(ID)
700   AENGY = AENGY + B(I)*B(I)*A(ID)                               ACT  47C
                              .
                              .
                              .
      GO TO 800                                                     ACT  61C
C**** ERROR MESSAGE AND STOP FOR SINGULAR SYSTEM.
 1100 WRITE(6,6000)INDEX
 6000 FORMAT(46H0ZERO DIAGONAL IN PARTIALLY REDUCED SYSTEM AT ,
     *16HDIAGONAL NUMBER ,I6)
      STOP
      END                                                           ACT  62C
      SUBROUTINE PROFIL (JDIAG,ID,IX,NDF,NEN1,NAD)                  PRO   1C
                              .
                              .
                              .
      GO TO 40                                                      PRO  16C
C**** RESTRAINED (J.LE.1) OR TIED(J.GT.1) NODES
```

```
30     IF(J.LE.1)ID(I,N) = 0
       IF(J.GT.1)ID(I,N)=ID(I,MOD(J-1,NUMNP)+1)
40     CONTINUE                                                  PRO 18C
                              .
                              .
                              .
       END                                                       PRO 49C
       SUBROUTINE PRTDIS(ID,X,B,F,NDM,NDF)                       PRT  1C
                              .
                              .
                              .
       RETURN                                                    PRT 25C
C**** CHANGE TIME FORMAT FROM E13.5 TO E16.8.
2000   FORMAT(A1,20A4//5X,19HNODAL DISPLACEMENTS,5X,4HTIME,E16.8//
     1   6X,4HNODE,9(I7,A4,A2))                                  PRT 27C
       END                                                       PRT 28C
       SUBROUTINE SHAP2(S,T,SHP,IX,NEL)                          SHA  1C
                              .
                              .
                              .
       T2 = (1.-T*T)/2.                                          SHA  7C
       DO 100 I=5,9                                            CR 3/78C
       DO 100 J = 1,3                                            SHA  9C
                              .
                              .
                              .
       IF(IX(9).EQ.0) GO TO 107                                  SHA 33C
C**** FROM R.L. TAYLOR -- MULTIPLY BY 4.
       SHP(1,9) = -4.*S*T2                                       SHA 34C
       SHP(2,9) = -4.*T*S2                                       SHA 35C
       SHP(3,9) = 4.*S2*T2                                       SHA 36C
                              .
                              .
                              .
       END                                                       SHA 51C
```

APPENDIX E: SAMPLE RUN

The following sample run demonstrates input and output associated with going from time 1981.150 to time 1981.275 by using calving law constant $a = 1165625$. The corresponding plots can be found in figure 4. During the actual modeling runs, 3 years were run at a time; the only modification necessary for such a multiple run is to change the macroinstruction LOOP*1 to LOOP*24. The asterisks appearing in some of the macros are merely spacers so that the subroutine FREEFM will place the constants that follow them in the proper card position. The input data contained in data set $$$$$$$.COLDATA is simply the freeform version of the data as listed at the beginning of the output, with single spaces between data items and an appended card with ** in columns 1 and 2. In addition, the title card has an asterisk in column 1 to indicate alphanumeric data. The general purpose program is contained in subroutine library $$$$$$$.LOADLIB, with the element subroutine ELMT04 for glacier flow in element ICE2 of that library. The data sets $$$$$$$.OLDSTART and $$$$$$$.RESTART are restart files read by macro REST and written by macro SAVE, respectively. Data set $$$$$$$.COLPLOT is the plot file. SYS1.RS232.DATA100.CALCOMP is a library of Calcomp[1] software designed to drive a local Calcomp drum plotter. Step COPYPLOT copies the plot file to the appropriate output class K for local plotting.

[1]The use of the brand name in this report is for identification purposes only and does not imply endorsement by the U.S. Geological Survey.

```
                                              H A S P    J O B    L O G
$21.50.22 JOB 8770 -- $$$$$$$$ -- BEGINNING EXEC - INIT 7 - CLASS F
$21.51.25 JOB 8770 ENDED  001105L
---- HASP-II RE1 STATISTICS ----
      40 CARDS READ
   1,116 SYSOUT PRINT RECORDS
     335 SYSOUT PUNCH RECORDS
    1.06MINUTES ELAPSED   TIME
//$$$$$$$$ JOB ($$$$$$$$$,$$$$,$$,10,1000,,,N),'SIKONIA*R*',            J8770
// CLASS=F,MSGLEVEL=(2,0)
***ROUTE PRINT REMOTE241 RE1
***ROUTE PUNCH REMOTE21 RE1
//FEAPRUN EXEC FORTXCG,TIME=5,COND=(0,LT),
// PARM.FORT='NOMAP,NAME(FEAP),OPTIMIZE(2),',
// ULIB='$$$$$$$.LOADLIB',PARM.GO='NOMAP,EP=FEAP,SIZE=210K',
// REGION.GO=236K
//FORT.SYSPRINT DD SYSOUT=A
//FORT.SYSIN DD *
IEF142I - STEP WAS EXECUTED - COND CODE 0000
IEF373I STEP /FORT    / START 81037.2150
IEF374I STEP /FORT    / STOP  81037.2150 CPU   0MIN 00.17SEC MAIN 248K LCS   0K
CCD006I STEP /FORT    / I/O IN DD SEQ; 4A4:2       563:1       155:0
CCD006I STEP /FORT    / I/O IN DD SEQ; 563:0       4D1:1
CCD001I STEP /FORT    / MEMORY RESERVED  250K; ACCESSES: DISK       4/ TAPE       0
//GO.SYSLIB DD
// DD DSN=SYS1.RS232.DATA100.CALCOMP,DISP=SHR
// DD DSN=SYS1.FORTGLIB,DISP=SHR
// DD DSN=XTENT.LIB,DISP=SHR
//GO.SYSLINB DD DSN=$$$$$$$.LOADLIB(ICE2),DISP=SHR
//GO.FT06F001 DD SYSOUT=A
//GO.SYSIN DD UNIT=SYSDK,SPACE=(TRK,(34,34)),
// DCB=(RECFM=FB,LRECL=80,BLKSIZE=640)
```

```
//GO.FT04F001 DD DSN=$$$$$$$.COLDATA,DISP=SHR
//GO.FT99F001 DD DSN=$$$$$$$.COLPLOT,DISP=OLD
//GO.FT15F001 DD DSN=$$$$$$$.OLDSTART,DISP=SHR
//GO.FT16F001 DD DSN=$$$$$$$.RESTART,DISP=(OLD,KEEP,KEEP)
IEF142I - STEP WAS EXECUTED - COND CODE 0000


IEF373I STEP /GO      / START 81037.2150
IEF374I STEP /GO      / STOP  81037.2151 CPU   0MIN 12.14SEC MAIN 234K LCS   0K
CCD006I STEP /GO      / I/O IN DD SEQ; 4A3:1      634:260     314:232
CCD006I STEP /GO      / I/O IN DD SEQ; 314:0      314:0       563:3
CCD006I STEP /GO      / I/O IN DD SEQ; 634:10     155:89      4A4:46
CCD006I STEP /GO      / I/O IN DD SEQ; 4B0:0      4A5:0       746:42
CCD006I STEP /GO      / I/O IN DD SEQ; 740:42     741:4       746:102
CCD001I STEP /GO      / MEMORY RESERVED  236K; ACCESSES: DISK    831/ TAPE       0
//COPYPLOT EXEC PGM=IEBGENER,TIME=1,COND=(0,LT),REGION=104K
//SYSPRINT DD SYSOUT=A
//SYSIN DD DUMMY
//SYSUT1 DD DSN=$$$$$$$.COLPLOT,DISP=SHR
//SYSUT2 DD SYSOUT=(K,,WS81),DCB=(RECFM=U,BLKSIZE=80)
IEF142I - STEP WAS EXECUTED - COND CODE 0000
IEF373I STEP /COPYPLOT/ START 81037.2151
IEF374I STEP /COPYPLOT/ STOP  81037.2151 CPU   0MIN 00.22SEC MAIN  34K LCS   0K
CCD006I STEP /COPYPLOT/ I/O IN DD SEQ; 4A2:1      000:0       740:43
CCD006I STEP /COPYPLOT/ I/O IN DD SEQ; 4B0:19
CCD001I STEP /COPYPLOT/ MEMORY RESERVED  104K; ACCESSES: DISK     63/ TAPE       0
IEF375I  JOB /$$$$$$$$/ START 81037.2150
IEF376I  JOB /$$$$$$$$/ STOP  81037.2151 CPU   0MIN 12.53SEC
CCD007A  JOB /$$$$$$$$/ BASIC CHRG: FIXED    CPU    DSK-IO  TPE-IO KBYTE-HR  TOTAL
                                    .41    2.79     .80              .54    4.54
CCD007D  JOB /$$$$$$$$/ LOCAL CHRG: FORMS      PRINTING  CRDS-RD  CRDS-PU  TOTAL
                                                             .00              .00
CCD002I  JOB /$$$$$$$$/ TOTAL CHARGE RECORDED =      $4.54
CCD003I  JOB /$$$$$$$$/ US GEOLOGICAL SURVEY RE1 OS REL 218 TIME = 21.51.25 DATE =
                         02/06/81


LEVEL 2.3.0 (JUNE 78)        OS/360  FORTRAN H EXTENDED       DATE 81.037/21.50.27
REQUESTED OPTIONS: NOMAP,NAME(FEAP),OPTIMIZE(2),
OPTIONS IN EFFECT: NAME(FEAP) OPTIMIZE(2) LINECOUNT(54) SIZE(MAX) AUTODBL(NONE)
                   SOURCE EBCDIC NOLIST NODECK OBJECT NOMAP NOFORMAT GOSTMT NOXREF
                     NOALC NOANSF NOTERM IBM
                C     PROGRAM FEAPB(INPUT=101,OUTPUT=101,TAPE4=INPUT,TAPE5=101,
                C    1              TAPE6=OUTPUT)
                C     MASTER MINIFEM                                          IFEM   1C
                C.... SET PROGRAM CAPACITY * MAX MUST AGREE WITH
                        DIMENSION OF M                                        IFEM   2C
ISN 0002              COMMON M(20000)
ISN 0003              COMMON/PSIZE/ MAX                                       IFEM   4C
ISN 0004              MAX = 20000
ISN 0005              CALL FREEFM(M,M(11),M(21),M(31),M(111))
ISN 0006              CALL PCONTR                                             IFEM   6C
ISN 0007              STOP                                                    IFEM   7C
ISN 0008              END                                                     IFEM   8C
```

```
*OPTIONS IN EFFECT*NAME(FEAP) OPTIMIZE(2) LINECOUNT(54) SIZE(MAX) AUTODBL(NONE)
*OPTIONS IN EFFECT*SOURCE EBCDIC NOLIST NODECK OBJECT NOMAP NOFORMAT GOSTMT NOXREF
  NOALC NOANSF NOTERM IBM
*STATISTICS*     SOURCE STATEMENTS =        7, PROGRAM SIZE =       318,
  SUBPROGRAM NAME =  FEAP
*STATISTICS*  NO  DIAGNOSTICS GENERATED
****** END OF COMPILATION ******                 58K BYTES OF CORE NOT USED
                                                       OS/360 LOADER
OPTIONS USED - PRINT,NOMAP,NOLET,CALL,RES,NOTERM,SIZE=215040,NAME=**GO
               EP=FEAP
  TOTAL LENGTH   2EC28
  ENTRY ADDRESS 4BA810


FIXED FORMAT FORM OF DATA CARDS FROM FREE FORMAT INPUT
    FEAP  COLUMBIA GLACIER VERTICAL CENTERLINE PROFILE ********************
           232        36         1         2         3         9         0
    NOPR
    COOR
             1         1       0.0    465.00
             3         1       0.0   -121.40
             5         0       0.0   -268.00
             6         1       0.0   -268.00
             8         0       0.0   -278.00
             9         1    762.50    430.00
            11         1    762.50   -103.60
            13         0    762.50   -237.00
            14         1    762.50   -237.00
            16         0    762.50   -247.00
            17         1   1525.00    411.00
            19         1   1525.00   -119.40
            21         0   1525.00   -252.00
            22         1   1525.00   -252.00
            24         0   1525.00   -262.00
            25         1   2287.50    401.00
            27         1   2287.50   -102.20
            29         0   2287.50   -228.00
            30         1   2287.50   -228.00
            32         0   2287.50   -238.00
            33         1   3050.00    370.00
            35         1   3050.00    -81.20
            37         0   3050.00   -194.00
            38         1   3050.00   -194.00
            40         0   3050.00   -204.00
            41         1   3812.50    340.00
            43         1   3812.50    -76.80
            45         0   3812.50   -181.00
            46         1   3812.50   -181.00
            48         0   3812.50   -191.00
            49         1   4575.00    314.00
            51         1   4575.00   -108.40
```

53	0	4575.00	-214.00
54	1	4575.00	-214.00
56	0	4575.00	-224.00
57	1	5337.50	290.00
59	1	5337.50	-190.00
61	0	5337.50	-310.00
62	1	5337.50	-310.00
64	0	5337.50	-320.00
65	1	6100.00	275.00
67	1	6100.00	-242.60
69	0	6100.00	-372.00
70	1	6100.00	-372.00
72	0	6100.00	-382.00
73	1	6862.50	264.00
75	1	6862.50	-255.20
77	0	6862.50	-385.00
78	1	6862.50	-385.00
80	0	6862.50	-395.00
81	1	7625.00	248.00
83	1	7625.00	-249.60
85	0	7625.00	-374.00
86	1	7625.00	-374.00
88	0	7625.00	-384.00
89	1	8387.50	226.00
91	1	8387.50	-253.20
93	0	8387.50	-373.00
94	1	8387.50	-373.00
96	0	8387.50	-383.00
97	1	9150.00	210.00
99	1	9150.00	-271.60
101	0	9150.00	-392.00
102	1	9150.00	-392.00
104	0	9150.00	-402.00
105	1	9912.50	198.00
107	1	9912.50	-273.20
109	0	9912.50	-391.00
110	1	9912.50	-391.00
112	0	9912.50	-401.00
113	1	10675.00	178.00
115	1	10675.00	-278.00
117	0	10675.00	-392.00
118	1	10675.?0	-392.00
120	0	10675.00	-402.00
121	1	11056.25	167.00
123	1	11056.25	-280.20
125	0	11056.25	-392.00
126	1	11056.25	-392.00
128	0	11056.25	-402.00
129	1	11437.50	159.00
131	1	11437.50	-265.80
133	0	11437.50	-372.00
134	1	11437.50	-372.00
136	0	11437.50	-382.00
137	1	11818.75	152.00

139	1	11818.75	-245.60
141	0	11818.75	-345.00
142	1	11818.75	-345.00
144	0	11818.75	-355.00
145	1	12200.00	140.00
147	1	12200.00	-220.00
149	0	12200.00	-310.00
150	1	12200.00	-310.00
152	0	12200.00	-320.00
153	1	12581.25	129.00
155	1	12581.25	-195.00
157	0	12581.25	-276.00
158	1	12581.25	-276.00
160	0	12581.25	-286.00
161	1	12962.50	118.00
163	1	12962.50	-176.40
165	0	12962.50	-250.00
166	1	12962.50	-250.00
168	0	12962.50	-260.00
169	1	13153.12	113.00
171	1	13153.12	-163.00
173	0	13153.12	-232.00
174	1	13153.12	-232.00
176	0	13153.12	-242.00
177	1	13343.75	108.00
179	1	13343.75	-147.20
181	0	13343.75	-211.00
182	1	13343.75	-211.00
184	0	13343.75	-221.00
185	1	13534.37	100.00
187	1	13534.37	-128.80
189	0	13534.37	-186.00
190	1	13534.37	-186.00
192	0	13534.37	-196.00
193	1	13725.00	96.00
195	1	13725.00	-102.40
197	0	13725.00	-152.00
198	1	13725.00	-152.00
200	0	13725.00	-162.00
201	1	13915.62	96.00
203	1	13915.62	-80.80
205	0	13915.62	-125.00
206	1	13915.62	-125.00
208	0	13915.62	-135.00
209	1	14106.25	96.00
211	1	14106.25	-57.60
213	0	14106.25	-96.00
214	1	14106.25	-96.00
216	0	14106.25	-106.00
217	1	14296.87	96.00
219	1	14296.87	-20.80
221	0	14296.87	-50.00
222	1	14296.87	-50.00
224	0	14296.87	-60.00

225	1	14487.50	96.00
227	1	14487.50	19.20
229	0	14487.50	0.0
230	1	14487.50	0.0
232	0	14487.50	-10.00

ELEM

01	1	3	19	17	1	11	18
9	2	10	2				
03	1	8	24	22	6	16	23
14	7	15	0				
04	1	19	35	33	17	27	34
25	18	26	2				
06	1	24	40	38	22	32	39
30	23	31	0				
07	1	35	51	49	33	43	50
41	34	42	2				
09	1	40	56	54	38	48	55
46	39	47	0				
10	1	51	67	65	49	59	66
57	50	58	2				
12	1	56	72	70	54	64	71
62	55	63	0				
13	1	67	83	81	65	75	82
73	66	74	2				
15	1	72	88	86	70	80	87
78	71	79	0				
16	1	83	99	97	81	91	98
89	82	90	2				
18	1	88	104	102	86	96	103
94	87	95	0				
19	1	99	115	113	97	107	114
105	98	106	2				
21	1	104	120	118	102	112	119
110	103	111	0				
22	1	115	131	129	113	123	130
121	114	122	2				
24	1	120	136	134	118	128	135
126	119	127	0				
25	1	131	147	145	129	139	146
137	130	138	2				
27	1	136	152	150	134	144	151
142	135	143	0				
28	1	147	163	161	145	155	162
153	146	154	2				
30	1	152	168	166	150	160	167
158	151	159	0				
31	1	163	179	177	161	171	178
169	162	170	2				
33	1	168	184	182	166	176	183
174	167	175	0				
34	1	179	195	193	177	187	194
185	178	186	2				
36	1	184	200	198	182	192	199
190	183	191	0				

```
MATE
        1         4
     .140      3.00      193.        8.
BOUN
        8         8        -1        -1
      200         0         1         1
        1         0         1
        2         0         1
        3         0         1
        5         0         1
        6         0         1         5
        4         8         1         0        -1
      196         0         0         0         1
        7         8         1         0        -1
      199         0         0         0         1
       14         8        13        13
      198         0       197       197
      201         1        -1        -1        -1
      232         0         1         1         1
FORC
        1         0    950.00
        2         0    943.56
        3         0    846.88
        4         0    784.83         0     -2.67
        5         0    698.25
        6         0    698.25
        7         0    349.13         0       40.
       12         0         0         0     -3.42
       20         0         0         0     -3.98
       28         0         0         0     -4.45
       36         0         0         0     -4.90
       44         0         0         0     -5.41
       52         0         0         0     -5.98
       60         0         0         0     -6.30
       68         0         0         0     -6.32
       76         0         0         0     -6.29
       84         0         0         0     -6.26
       92         0         0         0     -6.27
      100         0         0         0     -6.36
      108         0         0         0     -6.53
      116         0         0         0     -6.68
      124         0         0         0     -6.76
      132         0         0         0     -6.91
      140         0         0         0     -7.09
      148         0         0         0     -7.21
      156         0         0         0     -7.38
      164         0         0         0     -7.63
      172         0         0         0     -7.77
      180         0         0         0     -7.87
      188         0         0         0     -7.93
      196         0         0         0     -7.94
      204         0         0         0     -7.94
      212         0         0         0     -7.94
      220         0         0         0     -7.94
```

```
          228        0        0        0    -7.94
           15        0        0        0      37.
           23        0        0        0    25.14
           31        0        0        0     8.99
           39        0        0        0     8.38
           47        0        0        0     6.91
           55        0        0        0     8.63
           63        0        0        0    11.24
           71        0        0        0     7.51
           79        0        0        0     7.36
           87        0        0        0    15.91
           95        0        0        0    21.35
          103        0        0        0    22.58
          111        0        0        0    18.01
          119        0        0        0    20.84
          127        0        0        0      28.
          135        0        0        0      33.
          143        0        0        0    37.76
          151        0        0        0    40.25
          159        0        0        0    40.25
          167        0        0        0    40.25
          175        0        0        0    40.25
          183        0        0        0    40.25
          191        0        0        0    40.25
          199        0        0        0    40.25
          207        0        0        0    40.25
          215        0        0        0    40.25
          223        0        0        0    40.25
          231        0        0        0    40.25
TEMP
            6        0      950
           14        0     1030
           22        0      960
           30        0      922
           38        0      935
           46        0      930
           54        0      900
           62        0      839
           70        0      808
           78        0      812
           86        0      866
           94        0      904
          102        0      933
          110        0     1012
          118        0     1102
          126        0     1150
          134        0     1210
          142        0     1320
          150        0     1460
END
MACR
TOL                *    1.E-2
DT                 *    81.15
TIME
```

```
    REST                *        1.
    LOOP                *       10.
    TANF
    SOLV
    CONV
    DT                  *        0.
    TIME
    NEXT
    PLOT
    DISP
    SAVE
    LOOP                *        1
    LOOP                *        5
    DT                  *      .025
    UPNO
    TIME
    LOOP                *       10
    TANF
    SOLV
    CONV
    DT                  *        0.
    TIME
    NEXT
    NEXT
    PLOT
    DISP
    SAVE
    NEXT
    END
    SCAL
         -500.    16000.    -1000.    1000.     635.    12700.     .08       90.
    END
    STOP
END OF DATA INPUT
FEAP  COLUMBIA GLACIER VERTICAL CENTERLINE PROFILE ********************
    NUMBER OF NODAL POINTS        =   232
    NUMBER OF ELEMENTS            =    36
    NUMBER OF MATERIAL SETS       =     1
    DIMENSION OF COORDINATE SPACE=      2
    DEGREE OF FREEDOMS/NODE       =     3
    NODES PER ELEMENT (MAXIMUM)   =     9
    EXTRA D.O.F. TO ELEMENT       =     0
FEAP  COLUMBIA GLACIER VERTICAL CENTERLINE PROFILE ********************
    MATERIAL PROPERTIES
    MATERIAL SET  1 FOR ELEMENT TYPE 4
    TWO DIMENSIONAL ICE FLOW ELEMENT
         VISCOS. A   = 0.14000E+00
         VISCOS. N   = 0.30000E+01
         CALVE NODE = 0.19300E+03
         NODES DEEP = 0.80000E+01
FEAP  COLUMBIA GLACIER VERTICAL CENTERLINE PROFILE ********************
    MACRO INSTRUCTIONS
```

```
   MACRO STATEMENT     VARIABLE 1     VARIABLE 2
        TOL            .10000E-01      .0
        DT             81.150          .0
        TIME           .0              .0
        REST           1.0000          .0
        LOOP           10.000          .0
        TANF           .0              .0
        SOLV           .0              .0
        CONV           .0              .0
        DT             .0              .0
        TIME           .0              .0
        NEXT           .0              .0
        PLOT           .0              .0
        DISP           .0              .0
        SAVE           .0              .0
        LOOP           1.0000          .0
        LOOP           5.0000          .0
        DT             .25000E-01      .0
        UPNO           .0              .0
        TIME           .0              .0
        LOOP           10.000          .0
        TANF           .0              .0
        SOLV           .0              .0
        CONV           .0              .0
        DT             .0              .0
        TIME           .0              .0
        NEXT           .0              .0
        NEXT           .0              .0
        PLOT           .0              .0
        DISP           .0              .0
        SAVE           .0              .0
        NEXT           .0              .0
        END            .0              .0
**MACRO INSTRUCTION   1 EXECUTED**  TOL        V1 =     .1000E-01  , V2 =     .0
**MACRO INSTRUCTION   2 EXECUTED**  DT         V1 =     81.15      , V2 =     .0
**MACRO INSTRUCTION   3 EXECUTED**  TIME       V1 =     .0         , V2 =     .0
**MACRO INSTRUCTION   4 EXECUTED**  REST       V1 =     1.000      , V2 =     .0
**MACRO INSTRUCTION   5 EXECUTED**  LOOP       V1 =     10.00      , V2 =     12.00
**MACRO INSTRUCTION   6 EXECUTED**  TANF       V1 =     .0         , V2 =     .0
   FORCE CONVERGENCE TEST
        RNMAX =     .39112       RN     =     .39112       TOL    =    .10000E-01
**MACRO INSTRUCTION   7 EXECUTED**  SOLV       V1 =     .0         , V2 =     .0
**MACRO INSTRUCTION   8 EXECUTED**  CONV       V1 =     .0         , V2 =     .0
   DISPLACEMENT CONVERGENCE TEST
        UNMAX =     13151.       UN     =     .26771E-01   TOL    =    .10000E-01
**MACRO INSTRUCTION   9 EXECUTED**  DT         V1 =     .0         , V2 =     .0
**MACRO INSTRUCTION  10 EXECUTED**  TIME       V1 =     .0         , V2 =     .0
**MACRO INSTRUCTION  11 EXECUTED**  NEXT       V1 =     10.00      , V2 =     6.000
**MACRO INSTRUCTION  12 EXECUTED**  PLOT       V1 =     .0         , V2 =     .0
**MACRO INSTRUCTION  13 EXECUTED**  DISP       V1 =     .0         , V2 =     .0
```

FEAP COLUMBIA GLACIER VERTICAL CENTERLINE PROFILE ******************** TIME 81.150

PROPORTIONAL LOAD 1.0000

FEAP COLUMBIA GLACIER VERTICAL CENTERLINE PROFILE ********************

NODAL DISPLACEMENTS TIME 0.81149994E+02

NODE	1 COORD	2 COORD	1 DISPL	2 DISPL	3 DISPL	3 Explanation
1	0.0	465.0000	0.9500E+03	0.2658E+02	0.1454E+01	pressure
2	0.0	171.8000	0.9436E+03	0.3344E+02	0.1315E+02	hdot
3	0.0	-121.4000	0.8469E+03	0.4342E+02	0.5157E+02	pressure
4	0.0	-194.7000	0.7848E+03	0.4757E+02	-0.2670E+01	bdot
5	0.0	-268.0000	0.6982E+03	0.6042E+02	0.6282E+02	pressure
6	0.0	-268.0000	0.6982E+03	0.6042E+02	0.6363E+02	pressure
7	0.0	-273.0000	0.3491E+03	0.3069E+02	0.4000E+02	Abed
8	0.0	-278.0000	0.0	0.0	0.6445E+02	pressure
9	768.9395	456.6458	0.9785E+03	-0.2287E+02	0.3865E+00	
10	768.9395	179.2141	0.9582E+03	-0.1728E+02	0.8934E+00	
11	768.9395	-98.2178	0.9041E+03	-0.7195E+01	0.4951E+02	
12	768.9395	-167.5757	0.8807E+03	-0.2426E+01	-0.3426E+01	
13	768.9395	-236.9341	0.8561E+03	0.3056E+01	0.6186E+02	
14	768.9395	-236.9341	0.8561E+03	0.3056E+01	0.6235E+02	
15	768.9395	-241.9341	0.4316E+03	0.1645E+01	0.3694E+02	
16	768.9395	-246.9341	0.0	0.0	0.6325E+02	
17	1537.8792	429.7080	0.9810E+03	-0.1349E+02	0.6445E+00	
18	1537.8792	157.0576	0.9806E+03	-0.6178E+01	0.4564E+00	
19	1537.8792	-115.5928	0.9235E+03	-0.7839E+00	0.4921E+02	
20	1537.8792	-183.7554	0.8882E+03	-0.3930E+01	-0.3989E+01	
21	1537.8792	-251.9182	0.8384E+03	-0.1038E+02	0.6127E+02	
22	1537.8792	-251.9182	0.8384E+03	-0.1038E+02	0.6072E+02	
23	1537.8792	-256.9182	0.4251E+03	-0.5466E+01	0.2490E+02	
24	1537.8792	-261.9182	0.0	0.0	0.6156E+02	
25	2306.8184	422.9641	0.1045E+04	-0.1377E+02	-0.6725E-01	
26	2306.8184	162.8738	0.1024E+04	0.1247E+02	0.2698E+01	
27	2306.8184	-97.2166	0.9461E+03	0.3053E+02	0.4666E+02	
28	2306.8184	-162.2393	0.8948E+03	0.3436E+02	-0.4462E+01	
29	2306.8184	-227.2621	0.8223E+03	0.3676E+02	0.5836E+02	
30	2306.8184	-227.2621	0.8223E+03	0.3676E+02	0.5858E+02	
31	2306.8184	-232.2621	0.4134E+03	0.1864E+02	0.8783E+01	
32	2306.8184	-237.2621	0.0	0.0	0.5948E+02	
33	3075.7583	397.1182	0.1138E+04	-0.3268E+02	0.9516E-01	
34	3075.7583	160.9837	0.1114E+04	-0.3918E+01	0.2955E+01	
35	3075.7583	-75.1506	0.1025E+04	0.2363E+02	0.4208E+02	
36	3075.7583	-134.1843	0.9871E+03	0.2973E+02	-0.4916E+01	
37	3075.7583	-193.2181	0.9433E+03	0.3318E+02	0.5281E+02	
38	3075.7583	-193.2181	0.9433E+03	0.3318E+02	0.5299E+02	
39	3075.7583	-198.2181	0.4804E+03	0.1690E+02	0.8344E+01	
40	3075.7583	-203.2181	0.0	0.0	0.5386E+02	
41	3844.6975	369.8826	0.1206E+04	-0.5064E+02	0.2668E+00	
42	3844.6975	149.3443	0.1182E+04	-0.3620E+02	0.2964E+01	
43	3844.6975	-71.1938	0.1100E+04	-0.1966E+02	0.3966E+02	
44	3844.6975	-126.3284	0.1039E+04	-0.1402E+02	-0.5433E+01	
45	3844.6975	-181.4632	0.9477E+03	-0.9526E+01	0.4966E+02	
46	3844.6975	-181.4632	0.9477E+03	-0.9526E+01	0.4974E+02	
47	3844.6975	-186.4632	0.4826E+03	-0.4756E+01	0.6918E+01	
48	3844.6975	-191.4632	0.0	0.0	0.5063E+02	
49	4613.6367	319.5710	0.1219E+04	-0.8927E+02	0.2130E+01	
50	4613.6367	104.8030	0.1196E+04	-0.1060E+03	-0.3566E+00	

FEAP COLUMBIA GLACIER VERTICAL CENTERLINE PROFILE ********************

NODAL DISPLACEMENTS TIME 0.81149994E+02

NODE	1 COORD	2 COORD	1 DISPL	2 DISPL	3 DISPL
51	4613.6367	-109.9651	0.1124E+04	-0.1178E+03	0.3881E+02
52	4613.6367	-163.6570	0.1078E+04	-0.1176E+03	-0.6002E+01
53	4613.6367	-217.3492	0.1009E+04	-0.1131E+03	0.4791E+02
54	4613.6367	-217.3492	0.1009E+04	-0.1131E+03	0.4783E+02
55	4613.6367	-222.3492	0.5209E+03	-0.5836E+02	0.8741E+01
56	4613.6367	-227.3492	0.0	0.0	0.4870E+02
57	5382.5742	281.2563	0.1089E+04	-0.1921E+02	0.2112E+01
58	5382.5742	42.9097	0.1076E+04	-0.5648E+02	0.6542E+01
59	5382.5742	-195.4370	0.9849E+03	-0.8647E+02	0.4367E+02
60	5382.5742	-255.0234	0.9346E+03	-0.9266E+02	-0.6310E+01
61	5382.5742	-314.6104	0.8710E+03	-0.9498E+02	0.5376E+02
62	5382.5742	-314.6104	0.8710E+03	-0.9498E+02	0.5342E+02
63	5382.5742	-319.6104	0.4392E+03	-0.4815E+02	0.1120E+02
64	5382.5742	-324.6104	0.0	0.0	0.5429E+02
65	6151.5156	270.0581	0.1001E+04	0.3958E+01	0.2001E+01
66	6151.5156	12.2664	0.9778E+03	-0.8922E+01	0.5873E+01
67	6151.5156	-245.5256	0.8962E+03	-0.2281E+02	0.4647E+02
68	6151.5156	-309.9736	0.8555E+03	-0.2435E+02	-0.6320E+01
69	6151.5156	-374.4216	0.8090E+03	-0.2707E+02	0.5796E+02
70	6151.5156	-374.4216	0.8090E+03	-0.2707E+02	0.5769E+02
71	6151.5156	-379.4216	0.4065E+03	-0.1364E+02	0.7387E+01
72	6151.5156	-384.4216	0.0	0.0	0.5858E+02
73	6920.4570	256.9724	0.9923E+03	-0.7751E+01	0.6951E+00
74	6920.4570	0.1812	0.9711E+03	-0.5736E+01	0.6039E+01
75	6920.4570	-256.6106	0.8839E+03	-0.1563E+00	0.4632E+02
76	6920.4570	-320.8083	0.8274E+03	0.8642E+00	-0.6288E+01
77	6920.4570	-385.0063	0.7512E+03	0.1675E+01	0.5767E+02
78	6920.4570	-385.0063	0.7512E+03	0.1675E+01	0.5769E+02
79	6920.4570	-390.0063	0.3774E+03	0.8385E+00	0.7704E+01
80	6920.4570	-395.0063	0.0	0.0	0.5857E+02
81	7689.3945	239.8603	0.1001E+04	-0.1462E+02	0.7923E+00
82	7689.3945	-5.4953	0.9865E+03	-0.1085E+02	0.5455E+01
83	7689.3945	-250.8506	0.9322E+03	0.1088E+01	0.4402E+02
84	7689.3945	-312.1895	0.8920E+03	0.8432E+01	-0.6259E+01
85	7689.3945	-373.5288	0.8343E+03	0.1788E+02	0.5492E+02
86	7689.3945	-373.5288	0.8343E+03	0.1788E+02	0.5547E+02
87	7689.3945	-378.5288	0.4222E+03	0.9346E+01	0.1649E+02
88	7689.3945	-383.5288	0.0	0.0	0.5638E+02
89	8458.3359	216.2448	0.1006E+04	-0.2367E+02	0.7183E+00
90	8458.3359	-19.8220	0.9992E+03	-0.2227E+02	0.1431E+01
91	8458.3359	-255.8888	0.9684E+03	-0.1869E+02	0.4247E+02
92	8458.3359	-314.9053	0.9475E+03	-0.1722E+02	-0.6275E+01
93	8458.3359	-373.9224	0.9187E+03	-0.1509E+02	0.5284E+02
94	8458.3359	-373.9224	0.9187E+03	-0.1509E+02	0.5299E+02
95	8458.3359	-378.9224	0.4685E+03	-0.7634E+01	0.2164E+02
96	8458.3359	-383.9224	0.0	0.0	0.5387E+02
97	9227.2734	193.8667	0.1005E+04	-0.1446E+02	0.8011E+00
98	9227.2734	-40.8036	0.1001E+04	-0.1389E+02	0.2497E+01
99	9227.2734	-275.4739	0.9639E+03	-0.1330E+02	0.4242E+02
100	9227.2734	-334.1414	0.9376E+03	-0.1393E+02	-0.6374E+01

FEAP COLUMBIA GLACIER VERTICAL CENTERLINE PROFILE ********************

NODAL DISPLACEMENTS TIME 0.81149994E+02

NODE	1 COORD	2 COORD	1 DISPL	2 DISPL	3 DISPL
101	9227.2734	-392.8093	0.9023E+03	-0.1475E+02	0.5275E+02
102	9227.2734	-392.8093	0.9023E+03	-0.1475E+02	0.5262E+02
103	9227.2734	-397.8093	0.4529E+03	-0.7455E+01	0.2238E+02
104	9227.2734	-402.8093	0.0	0.0	0.5350E+02
105	9996.2148	177.0835	0.1014E+04	-0.6122E+01	0.6530E+00
106	9996.2148	-50.1546	0.1008E+04	-0.3719E+01	0.1126E+02
107	9996.2148	-277.3923	0.9791E+03	0.1995E+00	0.4115E+02
108	9996.2148	-334.2019	0.9559E+03	0.1658E+01	-0.6547E+01
109	9996.2148	-391.0117	0.9225E+03	0.3707E+01	0.5117E+02
110	9996.2148	-391.0117	0.9225E+03	0.3707E+01	0.5135E+02
111	9996.2148	-396.0117	0.4677E+03	0.1906E+01	0.1796E+02
112	9996.2148	-401.0117	0.0	0.0	0.5223E+02
113	10765.1523	156.6192	0.1020E+04	-0.1305E+02	0.7924E+00
114	10765.1523	-62.8644	0.1015E+04	-0.1277E+02	0.1112E+02
115	10765.1523	-282.3479	0.9908E+03	-0.1292E+02	0.3919E+02
116	10765.1523	-337.2187	0.9737E+03	-0.1291E+02	-0.6705E+01
117	10765.1523	-392.0901	0.9501E+03	-0.1263E+02	0.4880E+02
118	10765.1523	-392.0901	0.9501E+03	-0.1263E+02	0.4882E+02
119	10765.1523	-397.0901	0.4841E+03	-0.6267E+01	0.2214E+02
120	10765.1523	-402.0901	0.0	0.0	0.4971E+02
121	11149.6211	149.5192	0.1024E+04	0.6732E+01	0.3306E+00
122	11149.6211	-65.8687	0.1026E+04	0.1787E+02	0.9019E+01
123	11149.6211	-281.2566	0.9940E+03	0.2880E+02	0.3899E+02
124	11149.6211	-335.1035	0.9772E+03	0.3101E+02	-0.6790E+01
125	11149.6211	-388.9509	0.9582E+03	0.3309E+02	0.4855E+02
126	11149.6211	-388.9509	0.9582E+03	0.3309E+02	0.4881E+02
127	11149.6211	-393.9509	0.4736E+03	0.1643E+02	0.2942E+02
128	11149.6211	-398.9509	0.0	0.0	0.4968E+02
129	11534.0937	148.0525	0.1061E+04	0.2465E+02	-0.7999E-01
130	11534.0937	-57.4969	0.1056E+04	0.4870E+02	0.2809E+02
131	11534.0937	-263.0461	0.1029E+04	0.6691E+02	0.3699E+02
132	11534.0937	-314.4336	0.1013E+04	0.7144E+02	-0.6953E+01
133	11534.0937	-365.8213	0.9896E+03	0.7523E+02	0.4636E+02
134	11534.0937	-365.8213	0.9896E+03	0.7523E+02	0.4682E+02
135	11534.0937	-370.8213	0.5074E+03	0.3856E+02	0.3423E+02
136	11534.0937	-375.8213	0.0	0.0	0.4772E+02
137	11918.5625	140.1805	0.1124E+04	0.1301E+02	-0.1882E+00
138	11918.5625	-50.5356	0.1108E+04	0.4891E+02	0.3065E+02
139	11918.5625	-241.2516	0.1066E+04	0.7625E+02	0.3374E+02
140	11918.5625	-288.9304	0.1050E+04	0.8179E+02	-0.7127E+01
141	11918.5625	-336.6099	0.1030E+04	0.8593E+02	0.4247E+02
142	11918.5625	-336.6099	0.1030E+04	0.8593E+02	0.4300E+02
143	11918.5625	-341.6099	0.5193E+03	0.4347E+02	0.3863E+02
144	11918.5625	-346.6099	0.0	0.0	0.4388E+02
145	12303.0312	131.3288	0.1210E+04	0.6849E+01	-0.2639E+00
146	12303.0312	-41.4878	0.1192E+04	0.5015E+02	0.3638E+02
147	12303.0312	-214.3042	0.1136E+04	0.9043E+02	0.3090E+02
148	12303.0312	-257.5083	0.1114E+04	0.9956E+02	-0.7251E+01
149	12303.0312	-300.7129	0.1088E+04	0.1066E+03	0.3895E+02
150	12303.0312	-300.7129	0.1088E+04	0.1066E+03	0.3971E+02

FEAP COLUMBIA GLACIER VERTICAL CENTERLINE PROFILE ********************

NODAL DISPLACEMENTS TIME 0.81149994E+02

NODE	1 COORD	2 COORD	1 DISPL	2 DISPL	3 DISPL
151	12303.0312	-305.7129	0.5562E+03	0.5478E+02	0.4050E+02
152	12303.0312	-310.7129	0.0	0.0	0.4061E+02
153	12687.5039	116.6096	0.1298E+04	-0.1562E+01	-0.2111E+00
154	12687.5039	-37.2140	0.1287E+04	0.3523E+02	0.4183E+02
155	12687.5039	-191.0374	0.1237E+04	0.7301E+02	0.2695E+02
156	12687.5039	-229.4932	0.1215E+04	0.8263E+02	-0.7442E+01
157	12687.5039	-267.9495	0.1189E+04	0.9161E+02	0.3394E+02
158	12687.5039	-267.9495	0.1189E+04	0.9161E+02	0.3488E+02
159	12687.5039	-272.9495	0.6078E+03	0.4715E+02	0.4025E+02
160	12687.5039	-277.9495	0.0	0.0	0.3576E+02
161	13071.9727	101.4698	0.1391E+04	0.1581E+02	-0.3078E+00
162	13071.9727	-34.5920	0.1386E+04	0.5316E+02	0.6919E+02
163	13071.9727	-170.6537	0.1342E+04	0.8741E+02	0.2428E+02
164	13071.9727	-204.6691	0.1325E+04	0.9566E+02	-0.7724E+01
165	13071.9727	-238.6848	0.1304E+04	0.1033E+03	0.3045E+02
166	13071.9727	-238.6848	0.1304E+04	0.1033E+03	0.3135E+02
167	13071.9727	-243.6848	0.6799E+03	0.5462E+02	0.4025E+02
168	13071.9727	-248.6848	0.0	0.0	0.3225E+02
169	13264.2031	94.3128	0.1473E+04	0.9351E+01	-0.5730E+00
170	13264.2031	-31.4634	0.1454E+04	0.7239E+02	0.3975E+02
171	13264.2031	-157.2396	0.1403E+04	0.1234E+03	0.2189E+02
172	13264.2031	-188.6836	0.1385E+04	0.1339E+03	-0.7833E+01
173	13264.2031	-220.1277	0.1363E+04	0.1437E+03	0.2756E+02
174	13264.2031	-220.1277	0.1363E+04	0.1437E+03	0.2865E+02
175	13264.2031	-225.1277	0.6702E+03	0.7095E+02	0.4025E+02
176	13264.2031	-230.1277	0.0	0.0	0.2952E+02
177	13456.4414	89.8647	0.1602E+04	-0.7009E+01	-0.1291E+01
178	13456.4414	-24.7626	0.1576E+04	0.1070E+03	0.6118E+02
179	13456.4414	-139.3900	0.1492E+04	0.1856E+03	0.2025E+02
180	13456.4414	-168.0467	0.1469E+04	0.2002E+03	-0.7910E+01
181	13456.4414	-196.7039	0.1450E+04	0.2093E+03	0.2610E+02
182	13456.4414	-196.7039	0.1450E+04	0.2093E+03	0.2713E+02
183	13456.4414	-201.7039	0.8329E+03	0.1207E+03	0.4025E+02
184	13456.4414	-206.7039	0.0	0.0	0.2811E+02
185	13648.6719	76.2207	0.1824E+04	-0.4276E+02	-0.8349E+00
186	13648.6719	-20.9452	0.1795E+04	0.6911E+02	0.6717E+02
187	13648.6719	-118.1111	0.1689E+04	0.1934E+03	0.1660E+02
188	13648.6719	-142.4025	0.1643E+04	0.2221E+03	-0.7942E+01
189	13648.6719	-166.6940	0.1591E+04	0.2464E+03	0.2117E+02
190	13648.6719	-166.6940	0.1591E+04	0.2464E+03	0.2295E+02
191	13648.6719	-171.6940	0.7263E+03	0.1141E+03	0.4025E+02
192	13648.6719	-176.6940	0.0	0.0	0.2379E+02
193	13840.9141	66.9960	0.1955E+04	-0.2757E+02	-0.1211E+01
194	13840.9141	-13.7014	0.2063E+04	0.4808E+02	0.1659E+02
195	13840.9141	-94.3988	0.2034E+04	0.1827E+03	0.1231E+02
196	13840.9141	-114.5732	0.2000E+04	0.2339E+03	-0.7941E+01
197	13840.9141	-134.7476	0.1959E+04	0.2749E+03	0.1806E+02
198	13840.9141	-134.7476	0.1959E+04	0.2749E+03	0.2018E+02
199	13840.9141	-139.7476	0.1570E+04	0.2394E+03	0.4025E+02
200	13840.9141	-144.7476	0.0	0.0	0.2173E+02

```
**MACRO INSTRUCTION  14 EXECUTED**  SAVE           V1 =      .0          , V2 =      .0
**MACRO INSTRUCTION  15 EXECUTED**  LOOP           V1 =      1.000       , V2 =      32.00
**MACRO INSTRUCTION  16 EXECUTED**  LOOP           V1 =      5.000       , V2 =      28.00
**MACRO INSTRUCTION  17 EXECUTED**  DT             V1 =      .2500E-01   , V2 =      .0
**MACRO INSTRUCTION  18 EXECUTED**  UPNO           V1 =      .0          , V2 =      .0
**MACRO INSTRUCTION  19 EXECUTED**  TIME           V1 =      .0          , V2 =      .0
**MACRO INSTRUCTION  20 EXECUTED**  LOOP           V1 =      10.00       , V2 =      27.00
**MACRO INSTRUCTION  21 EXECUTED**  TANF           V1 =      .0          , V2 =      .0
   FORCE CONVERGENCE TEST
        RNMAX =     1985.6         RN     =     1985.6           TOL    =    .10000E-01
**MACRO INSTRUCTION  22 EXECUTED**  SOLV           V1 =      .0          , V2 =      .0
**MACRO INSTRUCTION  23 EXECUTED**  CONV           V1 =      .0          , V2 =      .0
   DISPLACEMENT CONVERGENCE TEST
        UNMAX =     13008.         UN     =     166.76           TOL    =    .10000E-01
**MACRO INSTRUCTION  24 EXECUTED**  DT             V1 =      .0          , V2 =      .0
**MACRO INSTRUCTION  25 EXECUTED**  TIME           V1 =      .0          , V2 =      .0
**MACRO INSTRUCTION  26 EXECUTED**  NEXT           V1 =      1.000       , V2 =      21.00
**MACRO INSTRUCTION  21 EXECUTED**  TANF           V1 =      .0          , V2 =      .0
   FORCE CONVERGENCE TEST
        RNMAX =     15.457         RN     =     15.457           TOL    =    .10000E-01
**MACRO INSTRUCTION  22 EXECUTED**  SOLV           V1 =      .0          , V2 =      .0
**MACRO INSTRUCTION  23 EXECUTED**  CONV           V1 =      .0          , V2 =      .0
   DISPLACEMENT CONVERGENCE TEST
        UNMAX =     13010.         UN     =     2.4093           TOL    =    .10000E-01
**MACRO INSTRUCTION  24 EXECUTED**  DT             V1 =      .0          , V2 =      .0
**MACRO INSTRUCTION  25 EXECUTED**  TIME           V1 =      .0          , V2 =      .0
**MACRO INSTRUCTION  26 EXECUTED**  NEXT           V1 =      10.00       , V2 =      21.00
**MACRO INSTRUCTION  27 EXECUTED**  NEXT           V1 =      1.000       , V2 =      17.00
**MACRO INSTRUCTION  17 EXECUTED**  DT             V1 =      .2500E-01   , V2 =      .0
**MACRO INSTRUCTION  18 EXECUTED**  UPNO           V1 =      .0          , V2 =      .0
**MACRO INSTRUCTION  19 EXECUTED**  TIME           V1 =      .0          , V2 =      .0
**MACRO INSTRUCTION  20 EXECUTED**  LOOP           V1 =      10.00       , V2 =      27.00
**MACRO INSTRUCTION  21 EXECUTED**  TANF           V1 =      .0          , V2 =      .0
   FORCE CONVERGENCE TEST
        RNMAX =     2440.2         RN     =     2440.2           TOL    =    .10000E-01
**MACRO INSTRUCTION  22 EXECUTED**  SOLV           V1 =      .0          , V2 =      .0
**MACRO INSTRUCTION  23 EXECUTED**  CONV           V1 =      .0          , V2 =      .0
   DISPLACEMENT CONVERGENCE TEST
        UNMAX =     12846.         UN     =     195.34           TOL    =    .10000E-01
**MACRO INSTRUCTION  24 EXECUTED**  DT             V1 =      .0          , V2 =      .0
**MACRO INSTRUCTION  25 EXECUTED**  TIME           V1 =      .0          , V2 =      .0
**MACRO INSTRUCTION  26 EXECUTED**  NEXT           V1 =      1.000       , V2 =      21.00
**MACRO INSTRUCTION  21 EXECUTED**  TANF           V1 =      .0          , V2 =      .0
   FORCE CONVERGENCE TEST
        RNMAX =     19.483         RN     =     19.483           TOL    =    .10000E-01
**MACRO INSTRUCTION  22 EXECUTED**  SOLV           V1 =      .0          , V2 =      .0
**MACRO INSTRUCTION  23 EXECUTED**  CONV           V1 =      .0          , V2 =      .0
   DISPLACEMENT CONVERGENCE TEST
        UNMAX =     12848.         UN     =     3.0509           TOL    =    .10000E-01
**MACRO INSTRUCTION  24 EXECUTED**  DT             V1 =      .0          , V2 =      .0
**MACRO INSTRUCTION  25 EXECUTED**  TIME           V1 =      .0          , V2 =      .0
**MACRO INSTRUCTION  26 EXECUTED**  NEXT           V1 =      10.00       , V2 =      21.00
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**MACRO INSTRUCTION  27 EXECUTED**  NEXT          V1 =     2.000      , V2 =     17.00
**MACRO INSTRUCTION  17 EXECUTED**  DT            V1 =     .2500E-01  , V2 =     .0
**MACRO INSTRUCTION  18 EXECUTED**  UPNO          V1 =     .0         , V2 =     .0
**MACRO INSTRUCTION  19 EXECUTED**  TIME          V1 =     .0         , V2 =     .0
**MACRO INSTRUCTION  20 EXECUTED**  LOOP          V1 =     10.00      , V2 =     27.00
**MACRO INSTRUCTION  21 EXECUTED**  TANF          V1 =     .0         , V2 =     .0
   FORCE CONVERGENCE TEST
         RNMAX =     2394.2        RN     =    2394.2         TOL    =    .10000E-01
**MACRO INSTRUCTION  22 EXECUTED**  SOLV          V1 =     .0         , V2 =     .0
**MACRO INSTRUCTION  23 EXECUTED**  CONV          V1 =     .0         , V2 =     .0
   DISPLACEMENT CONVERGENCE TEST
         UNMAX =     12704.        UN     =    175.48         TOL    =    .10000E-01
**MACRO INSTRUCTION  24 EXECUTED**  DT            V1 =     .0         , V2 =     .0
**MACRO INSTRUCTION  25 EXECUTED**  TIME          V1 =     .0         , V2 =     .0
**MACRO INSTRUCTION  26 EXECUTED**  NEXT          V1 =     1.000      , V2 =     21.00
**MACRO INSTRUCTION  21 EXECUTED**  TANF          V1 =     .0         , V2 =     .0
   FORCE CONVERGENCE TEST
         RNMAX =     27.012        RN     =    27.012         TOL    =    .10000E-01
**MACRO INSTRUCTION  22 EXECUTED**  SOLV          V1 =     .0         , V2 =     .0
**MACRO INSTRUCTION  23 EXECUTED**  CONV          V1 =     .0         , V2 =     .0
   DISPLACEMENT CONVERGENCE TEST
         UNMAX =     12706.        UN     =    2.7596         TOL    =    .10000E-01
**MACRO INSTRUCTION  24 EXECUTED**  DT            V1 =     .0         , V2 =     .0
**MACRO INSTRUCTION  25 EXECUTED**  TIME          V1 =     .0         , V2 =     .0
**MACRO INSTRUCTION  26 EXECUTED**  NEXT          V1 =     10.00      , V2 =     21.00
**MACRO INSTRUCTION  27 EXECUTED**  NEXT          V1 =     3.000      , V2 =     17.00
**MACRO INSTRUCTION  17 EXECUTED**  DT            V1 =     .2500E-01  , V2 =     .0
**MACRO INSTRUCTION  18 EXECUTED**  UPNO          V1 =     .0         , V2 =     .0
**MACRO INSTRUCTION  19 EXECUTED**  TIME          V1 =     .0         , V2 =     .0
**MACRO INSTRUCTION  20 EXECUTED**  LOOP          V1 =     10.00      , V2 =     27.00
**MACRO INSTRUCTION  21 EXECUTED**  TANF          V1 =     .0         , V2 =     .0
   FORCE CONVERGENCE TEST
         RNMAX =     1750.1        RN     =    1750.1         TOL    =    .10000E-01
**MACRO INSTRUCTION  22 EXECUTED**  SOLV          V1 =     .0         , V2 =     .0
**MACRO INSTRUCTION  23 EXECUTED**  CONV          V1 =     .0         , V2 =     .0
   DISPLACEMENT CONVERGENCE TEST
         UNMAX =     12572.        UN     =    152.09         TOL    =    .10000E-01
**MACRO INSTRUCTION  24 EXECUTED**  DT            V1 =     .0         , V2 =     .0
**MACRO INSTRUCTION  25 EXECUTED**  TIME          V1 =     .0         , V2 =     .0
**MACRO INSTRUCTION  26 EXECUTED**  NEXT          V1 =     1.000      , V2 =     21.00
**MACRO INSTRUCTION  21 EXECUTED**  TANF          V1 =     .0         , V2 =     .0
   FORCE CONVERGENCE TEST
         RNMAX =     127.53        RN     =    127.53         TOL    =    .10000E-01
**MACRO INSTRUCTION  22 EXECUTED**  SOLV          V1 =     .0         , V2 =     .0
**MACRO INSTRUCTION  23 EXECUTED**  CONV          V1 =     .0         , V2 =     .0
   DISPLACEMENT CONVERGENCE TEST
         UNMAX =     12574.        UN     =    2.6256         TOL    =    .10000E-01
**MACRO INSTRUCTION  24 EXECUTED**  DT            V1 =     .0         , V2 =     .0
**MACRO INSTRUCTION  25 EXECUTED**  TIME          V1 =     .0         , V2 =     .0
**MACRO INSTRUCTION  26 EXECUTED**  NEXT          V1 =     10.00      , V2 =     21.00
**MACRO INSTRUCTION  27 EXECUTED**  NEXT          V1 =     4.000      , V2 =     17.00
**MACRO INSTRUCTION  17 EXECUTED**  DT            V1 =     .2500E-01  , V2 =     .0
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 **MACRO INSTRUCTION  18 EXECUTED**  UPNO         V1 =     .0         , V2 =     .0
 **MACRO INSTRUCTION  19 EXECUTED**  TIME         V1 =     .0         , V2 =     .0
 **MACRO INSTRUCTION  20 EXECUTED**  LOOP         V1 =     10.00      , V2 =     27.00
 **MACRO INSTRUCTION  21 EXECUTED**  TANF         V1 =     .0         , V2 =     .0
   FORCE CONVERGENCE TEST
        RNMAX =      1291.0         RN    =     1291.0        TOL    =     .10000E-01
 **MACRO INSTRUCTION  22 EXECUTED**  SOLV         V1 =     .0         , V2 =     .0
 **MACRO INSTRUCTION  23 EXECUTED**  CONV         V1 =     .0         , V2 =     .0
   DISPLACEMENT CONVERGENCE TEST
        UNMAX =      12466.         UN    =     123.86        TOL    =     .10000E-01
 **MACRO INSTRUCTION  24 EXECUTED**  DT           V1 =     .0         , V2 =     .0
 **MACRO INSTRUCTION  25 EXECUTED**  TIME         V1 =     .0         , V2 =     .0
 **MACRO INSTRUCTION  26 EXECUTED**  NEXT         V1 =     10.00      , V2 =     21.00
 **MACRO INSTRUCTION  27 EXECUTED**  NEXT         V1 =     5.000      , V2 =     17.00
 **MACRO INSTRUCTION  28 EXECUTED**  PLOT         V1 =     .0         , V2 =     .0
 **MACRO INSTRUCTION  29 EXECUTED**  DISP         V1 =     .0         , V2 =     .0
FEAP  COLUMBIA GLACIER VERTICAL CENTERLINE PROFILE ******************** TIME   81.275
   PROPORTIONAL LOAD   1.0000
FEAP  COLUMBIA GLACIER VERTICAL CENTERLINE PROFILE ********************
   NODAL DISPLACEMENTS      TIME  0.81274963E+02
```

NODE	1 COORD	2 COORD	1 DISPL	2 DISPL	3 DISPL
1	0.0	465.0000	0.9500E+03	0.2882E+02	0.1535E+01
2	0.0	171.8000	0.9436E+03	0.3435E+02	0.1517E+02
3	0.0	-121.4000	0.8469E+03	0.4353E+02	0.5159E+02
4	0.0	-194.7000	0.7848E+03	0.4749E+02	-0.2670E+01
5	0.0	-268.0000	0.6982E+03	0.6017E+02	0.6284E+02
6	0.0	-268.0000	0.6982E+03	0.6017E+02	0.6365E+02
7	0.0	-273.0000	0.3491E+03	0.3056E+02	0.4000E+02
8	0.0	-278.0000	0.0	0.0	0.6447E+02
9	774.9919	456.7146	0.9753E+03	-0.2072E+02	0.4242E+00
10	774.9919	179.2788	0.9557E+03	-0.1599E+02	0.2853E+01
11	774.9919	-98.1570	0.9024E+03	-0.6765E+01	0.4952E+02
12	774.9919	-167.5159	0.8791E+03	-0.2233E+01	-0.3431E+01
13	774.9919	-236.8751	0.8548E+03	0.3018E+01	0.6187E+02
14	774.9919	-236.8751	0.8548E+03	0.3018E+01	0.6234E+02
15	774.9919	-241.8751	0.4309E+03	0.1622E+01	0.3688E+02
16	774.9919	-246.8751	0.0	0.0	0.6324E+02
17	1549.9841	429.6428	0.9762E+03	-0.1030E+02	0.6830E+00
18	1549.9841	157.0532	0.9760E+03	-0.3824E+01	0.3248E+01
19	1549.9841	-115.5364	0.9192E+03	0.5947E+00	0.4924E+02
20	1549.9841	-183.6838	0.8838E+03	-0.2891E+01	-0.3997E+01
21	1549.9841	-251.8313	0.8338E+03	-0.9678E+01	0.6130E+02
22	1549.9841	-251.8313	0.8338E+03	-0.9678E+01	0.6073E+02
23	1549.9841	-256.8313	0.4227E+03	-0.5113E+01	0.2468E+02
24	1549.9841	-261.8313	0.0	0.0	0.6157E+02
25	2324.9758	423.0266	0.1037E+04	-0.1124E+02	-0.5320E-01
26	2324.9758	163.1912	0.1017E+04	0.1365E+02	0.5053E+01
27	2324.9758	-96.6445	0.9395E+03	0.3074E+02	0.4662E+02
28	2324.9758	-161.6035	0.8878E+03	0.3431E+02	-0.4473E+01
29	2324.9758	-226.5626	0.8145E+03	0.3651E+02	0.5830E+02
30	2324.9758	-226.5626	0.8145E+03	0.3651E+02	0.5852E+02
31	2324.9758	-231.5626	0.4095E+03	0.1851E+02	0.8597E+01
32	2324.9758	-236.5626	0.0	0.0	0.5942E+02
33	3099.9685	396.8281	0.1126E+04	-0.2900E+02	0.1372E+00

34	3099.9685	161.0949	0.1103E+04	-0.2955E+01	0.6358E+01
35	3099.9685	-74.6382	0.1017E+04	0.2284E+02	0.4203E+02
36	3099.9685	-133.5715	0.9795E+03	0.2885E+02	-0.4932E+01
37	3099.9685	-192.5051	0.9362E+03	0.3245E+02	0.5274E+02
38	3099.9685	-192.5051	0.9362E+03	0.3245E+02	0.5293E+02
39	3099.9685	-197.5051	0.4769E+03	0.1656E+02	0.8310E+01
40	3099.9685	-202.5051	0.0	0.0	0.5380E+02
41	3874.9602	368.8999	0.1185E+04	-0.4704E+02	0.3037E+00
42	3874.9602	148.5506	0.1162E+04	-0.3464E+02	0.6679E+01
43	3874.9602	-71.7986	0.1084E+04	-0.2039E+02	0.3967E+02
44	3874.9602	-126.8860	0.1026E+04	-0.1568E+02	-0.5454E+01
45	3874.9602	-181.9735	0.9358E+03	-0.1186E+02	0.4961E+02
46	3874.9602	-181.9735	0.9358E+03	-0.1186E+02	0.4968E+02
47	3874.9602	-186.9735	0.4768E+03	-0.5977E+01	0.6931E+01
48	3874.9602	-191.9735	0.0	0.0	0.5056E+02
49	4649.9492	317.6731	0.1192E+04	-0.8114E+02	0.2195E+01
50	4649.9492	102.3462	0.1170E+04	-0.1003E+03	0.3837E+01

FEAP COLUMBIA GLACIER VERTICAL CENTERLINE PROFILE ********************

NODAL DISPLACEMENTS TIME 0.81274963E+02

NODE	1 COORD	2 COORD	1 DISPL	2 DISPL	3 DISPL
51	4649.9492	-112.9807	0.1094E+04	-0.1151E+03	0.3902E+02
52	4649.9492	-166.8125	0.1048E+04	-0.1162E+03	-0.6023E+01
53	4649.9492	-220.6443	0.9791E+03	-0.1126E+03	0.4809E+02
54	4649.9492	-220.6443	0.9791E+03	-0.1126E+03	0.4797E+02
55	4649.9492	-225.6443	0.5050E+03	-0.5810E+02	0.8847E+01
56	4649.9492	-230.6443	0.0	0.0	0.4884E+02
57	5424.9414	281.0791	0.1054E+04	-0.9754E+01	0.2136E+01
58	5424.9414	41.1132	0.1039E+04	-0.4953E+02	0.1298E+02
59	5424.9414	-198.8525	0.9476E+03	-0.8097E+02	0.4398E+02
60	5424.9414	-258.8440	0.8986E+03	-0.8734E+02	-0.6318E+01
61	5424.9414	-318.8357	0.8368E+03	-0.8974E+02	0.5414E+02
62	5424.9414	-318.8357	0.8368E+03	-0.8974E+02	0.5380E+02
63	5424.9414	-323.8357	0.4217E+03	-0.4547E+02	0.1113E+02
64	5424.9414	-328.8357	0.0	0.0	0.5467E+02
65	6199.9336	270.1453	0.9592E+03	0.5920E+01	0.2010E+01
66	6199.9336	11.4897	0.9382E+03	-0.6883E+01	0.8312E+01
67	6199.9336	-247.1658	0.8640E+03	-0.1952E+02	0.4661E+02
68	6199.9336	-311.8296	0.8260E+03	-0.2080E+02	-0.6319E+01
69	6199.9336	-376.4937	0.7824E+03	-0.2302E+02	0.5812E+02
70	6199.9336	-376.4937	0.7824E+03	-0.2302E+02	0.5789E+02
71	6199.9336	-381.4937	0.3934E+03	-0.1161E+02	0.7286E+01
72	6199.9336	-386.4937	0.0	0.0	0.5878E+02
73	6974.9297	256.8132	0.9498E+03	-0.3999E+01	0.8032E+00
74	6974.9297	0.1335	0.9307E+03	-0.2694E+01	0.9165E+01
75	6974.9297	-256.5461	0.8516E+03	0.1483E+01	0.4629E+02
76	6974.9297	-320.7161	0.7993E+03	0.2469E+01	-0.6286E+01
77	6974.9297	-384.8862	0.7284E+03	0.2891E+01	0.5766E+02
78	6974.9297	-384.8862	0.7284E+03	0.2891E+01	0.5767E+02
79	6974.9297	-389.8862	0.3661E+03	0.1450E+01	0.8074E+01
80	6974.9297	-394.8862	0.0	0.0	0.5855E+02
81	7749.9180	239.0933	0.9555E+03	-0.1150E+02	0.1017E+01
82	7749.9180	-5.8043	0.9420E+03	-0.9521E+01	0.8511E+01
83	7749.9180	-250.7019	0.8951E+03	0.1285E+00	0.4395E+02
84	7749.9180	-311.9260	0.8576E+03	0.7086E+01	-0.6259E+01

85	7749.9180	-373.1509	0.8030E+03	0.1561E+02	0.5488E+02
86	7749.9180	-373.1509	0.8030E+03	0.1561E+02	0.5538E+02
87	7749.9180	-378.1509	0.4069E+03	0.8168E+01	0.1701E+02
88	7749.9180	-383.1509	0.0	0.0	0.5629E+02
89	8524.9102	214.6174	0.9552E+03	-0.1915E+02	0.9557E+00
90	8524.9102	-21.2081	0.9497E+03	-0.1881E+02	0.4678E+01
91	8524.9102	-257.0334	0.9224E+03	-0.1738E+02	0.4242E+02
92	8524.9102	-315.9897	0.9030E+03	-0.1636E+02	-0.6280E+01
93	8524.9102	-374.9465	0.8762E+03	-0.1495E+02	0.5280E+02
94	8524.9102	-374.9465	0.8762E+03	-0.1495E+02	0.5290E+02
95	8524.9102	-379.9465	0.4469E+03	-0.7574E+01	0.2188E+02
96	8524.9102	-384.9465	0.0	0.0	0.5378E+02
97	9299.9023	192.5049	0.9525E+03	-0.8651E+01	0.1247E+01
98	9299.9023	-41.8501	0.9479E+03	-0.9233E+01	0.4868E+01
99	9299.9023	-276.2048	0.9116E+03	-0.1030E+02	0.4240E+02
100	9299.9023	-334.7937	0.8873E+03	-0.1063E+02	-0.6387E+01

FEAP COLUMBIA GLACIER VERTICAL CENTERLINE PROFILE ********************

NODAL DISPLACEMENTS TIME 0.81274963E+02

NODE	1 COORD	2 COORD	1 DISPL	2 DISPL	3 DISPL
101	9299.9023	-393.3828	0.8549E+03	-0.1176E+02	0.5282E+02
102	9299.9023	-393.3828	0.8549E+03	-0.1176E+02	0.5264E+02
103	9299.9023	-398.3828	0.4285E+03	-0.5936E+01	0.2214E+02
104	9299.9023	-403.3828	0.0	0.0	0.5353E+02
105	10074.8945	176.7471	0.9544E+03	-0.2155E+01	0.8901E+00
106	10074.8945	-50.3697	0.9492E+03	-0.1550E+01	0.1362E+02
107	10074.8945	-277.4863	0.9231E+03	0.6115E+00	0.4112E+02
108	10074.8945	-334.2656	0.9017E+03	0.1667E+01	-0.6564E+01
109	10074.8945	-391.0452	0.8706E+03	0.3158E+01	0.5113E+02
110	10074.8945	-391.0452	0.8706E+03	0.3158E+01	0.5127E+02
111	10074.8945	-396.0452	0.4419E+03	0.1616E+01	0.1799E+02
112	10074.8945	-401.0452	0.0	0.0	0.5215E+02
113	10849.8867	155.4968	0.9552E+03	-0.3793E+01	0.1113E+01
114	10849.8867	-63.5514	0.9507E+03	-0.4707E+01	0.1871E+02
115	10849.8867	-282.5994	0.9269E+03	-0.6922E+01	0.3919E+02
116	10849.8867	-337.3613	0.9104E+03	-0.7465E+01	-0.6725E+01
117	10849.8867	-392.1238	0.8885E+03	-0.7910E+01	0.4878E+02
118	10849.8867	-392.1238	0.8885E+03	-0.7910E+01	0.4873E+02
119	10849.8867	-397.1238	0.4510E+03	-0.3895E+01	0.2359E+02
120	10849.8867	-402.1238	0.0	0.0	0.4963E+02
121	11237.3828	150.2279	0.9566E+03	0.1928E+02	0.4064E+00
122	11237.3828	-63.8597	0.9571E+03	0.2765E+02	0.1498E+02
123	11237.3828	-277.9473	0.9271E+03	0.3572E+02	0.3892E+02
124	11237.3828	-331.4692	0.9118E+03	0.3718E+02	-0.6823E+01
125	11237.3828	-384.9915	0.8948E+03	0.3876E+02	0.4837E+02
126	11237.3828	-384.9915	0.8948E+03	0.3876E+02	0.4863E+02
127	11237.3828	-389.9915	0.4419E+03	0.1922E+02	0.3064E+02
128	11237.3828	-394.9915	0.0	0.0	0.4951E+02
129	11624.8789	150.7312	0.9885E+03	0.2897E+02	-0.4567E-01
130	11624.8789	-53.4030	0.9822E+03	0.5029E+02	0.3111E+02
131	11624.8789	-257.5371	0.9575E+03	0.6617E+02	0.3661E+02
132	11624.8789	-308.5706	0.9423E+03	0.7013E+02	-0.6995E+01
133	11624.8789	-359.6042	0.9215E+03	0.7316E+02	0.4593E+02
134	11624.8789	-359.6042	0.9215E+03	0.7316E+02	0.4634E+02
135	11624.8789	-364.6042	0.4721E+03	0.3746E+02	0.3537E+02

136	11624.8789	-369.6042	0.0	0.0	0.4724E+02
137	12012.3750	142.2204	0.1045E+04	0.1843E+02	-0.1379E+00
138	12012.3750	-45.9574	0.1031E+04	0.4992E+02	0.3542E+02
139	12012.3750	-234.1351	0.9915E+03	0.7445E+02	0.3338E+02
140	12012.3750	-281.1792	0.9765E+03	0.7946E+02	-0.7158E+01
141	12012.3750	-328.2241	0.9587E+03	0.8321E+02	0.4200E+02
142	12012.3750	-328.2241	0.9587E+03	0.8321E+02	0.4252E+02
143	12012.3750	-333.2241	0.4834E+03	0.4212E+02	0.3931E+02
144	12012.3750	-338.2241	0.0	0.0	0.4341E+02
145	12399.8711	133.1550	0.1119E+04	0.1276E+02	-0.1695E+00
146	12399.8711	-36.9271	0.1104E+04	0.4695E+02	0.4065E+02
147	12399.8711	-207.0090	0.1056E+04	0.7955E+02	0.3036E+02
148	12399.8711	-249.5296	0.1036E+04	0.8724E+02	-0.7293E+01
149	12399.8711	-292.0505	0.1012E+04	0.9337E+02	0.3825E+02
150	12399.8711	-292.0505	0.1012E+04	0.9337E+02	0.3897E+02

FEAP COLUMBIA GLACIER VERTICAL CENTERLINE PROFILE ********************

NODAL DISPLACEMENTS TIME 0.81274963E+02

NODE	1 COORD	2 COORD	1 DISPL	2 DISPL	3 DISPL
151	12399.8711	-297.0505	0.5201E+03	0.4817E+02	0.4056E+02
152	12399.8711	-302.0505	0.0	0.0	0.3987E+02
153	12787.3672	118.2029	0.1189E+04	0.6555E+01	-0.1346E+00
154	12787.3672	-33.4582	0.1181E+04	0.3665E+02	0.4326E+02
155	12787.3672	-185.1193	0.1139E+04	0.6797E+02	0.2667E+02
156	12787.3672	-223.0344	0.1121E+04	0.7596E+02	-0.7505E+01
157	12787.3672	-260.9500	0.1099E+04	0.8338E+02	0.3354E+02
158	12787.3672	-260.9500	0.1099E+04	0.8338E+02	0.3440E+02
159	12787.3672	-265.9500	0.5602E+03	0.4281E+02	0.4025E+02
160	12787.3672	-270.9500	0.0	0.0	0.3529E+02
161	13174.8633	104.6301	0.1271E+04	0.2728E+02	-0.3111E+00
162	13174.8633	-29.1245	0.1270E+04	0.6326E+02	0.6827E+02
163	13174.8633	-162.8789	0.1233E+04	0.9604E+02	0.2384E+02
164	13174.8633	-196.3176	0.1218E+04	0.1039E+03	-0.7783E+01
165	13174.8633	-229.7564	0.1199E+04	0.1111E+03	0.2995E+02
166	13174.8633	-229.7564	0.1199E+04	0.1111E+03	0.3087E+02
167	13174.8633	-234.7564	0.6223E+03	0.5801E+02	0.4025E+02
168	13174.8633	-239.7564	0.0	0.0	0.3177E+02
169	13368.6094	97.0733	0.1357E+04	0.2381E+02	-0.5945E+00
170	13368.6094	-24.9426	0.1339E+04	0.8648E+02	0.5714E+02
171	13368.6094	-146.9585	0.1291E+04	0.1334E+03	0.2130E+02
172	13368.6094	-177.4623	0.1274E+04	0.1423E+03	-0.7880E+01
173	13368.6094	-207.9665	0.1257E+04	0.1504E+03	0.2677E+02
174	13368.6094	-207.9665	0.1257E+04	0.1504E+03	0.2781E+02
175	13368.6094	-212.9665	0.6225E+03	0.7506E+02	0.4025E+02
176	13368.6094	-217.9665	0.0	0.0	0.2869E+02
177	13562.3594	92.0420	0.1496E+04	-0.5806E-01	-0.1345E+01
178	13562.3594	-17.4035	0.1464E+04	0.1197E+03	0.5321E+02
179	13562.3594	-126.8490	0.1375E+04	0.2031E+03	0.1920E+02
180	13562.3594	-154.2104	0.1351E+04	0.2184E+03	-0.7935E+01
181	13562.3594	-181.5718	0.1331E+04	0.2276E+03	0.2491E+02
182	13562.3594	-181.5718	0.1331E+04	0.2276E+03	0.2601E+02
183	13562.3594	-186.5718	0.7400E+03	0.1243E+03	0.4025E+02
184	13562.3594	-191.5718	0.0	0.0	0.2697E+02
185	13756.1055	78.2083	0.1743E+04	-0.4757E+02	-0.9306E+00
186	13756.1055	-11.9215	0.1707E+04	0.6358E+02	0.8268E+02

187	13756.1055	-102.0513	0.1595E+04	0.1812E+03	0.1544E+02
188	13756.1055	-124.5837	0.1548E+04	0.2081E+03	-0.7941E+01
189	13756.1055	-147.1162	0.1495E+04	0.2310E+03	0.1974E+02
190	13756.1055	-147.1162	0.1495E+04	0.2310E+03	0.2155E+02
191	13756.1055	-152.1162	0.7126E+03	0.1115E+03	0.4025E+02
192	13756.1055	-157.1162	0.0	0.0	0.2240E+02
193	13949.8594	62.7661	0.1895E+04	-0.7402E+02	-0.1078E+01
194	13949.8594	-10.3158	0.1965E+04	-0.1467E+02	0.6399E+02
195	13949.8594	-83.3978	0.1922E+04	0.9215E+02	0.1077E+02
196	13949.8594	-101.6682	0.1889E+04	0.1333E+03	-0.7940E+01
197	13949.8594	-119.9387	0.1850E+04	0.1682E+03	0.1576E+02
198	13949.8594	-119.9387	0.1850E+04	0.1682E+03	0.1780E+02
199	13949.8594	-124.9387	0.1454E+04	0.1565E+03	0.4025E+02
200	13949.8594	-129.9387	0.0	0.0	0.1928E+02

```
**MACRO INSTRUCTION  30 EXECUTED**  SAVE     V1 =      .0      , V2 =     .0
**MACRO INSTRUCTION  31 EXECUTED**  NEXT     V1 =     1.000    , V2 =   16.00
DATA SET UTILITY - GENERATE                                       PAGE 0001
PROCESSING ENDED AT EOD
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